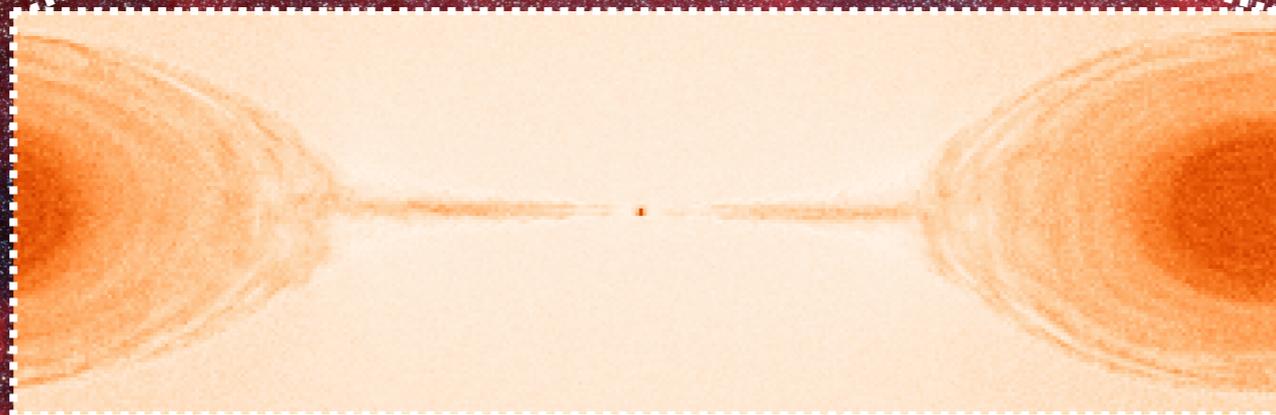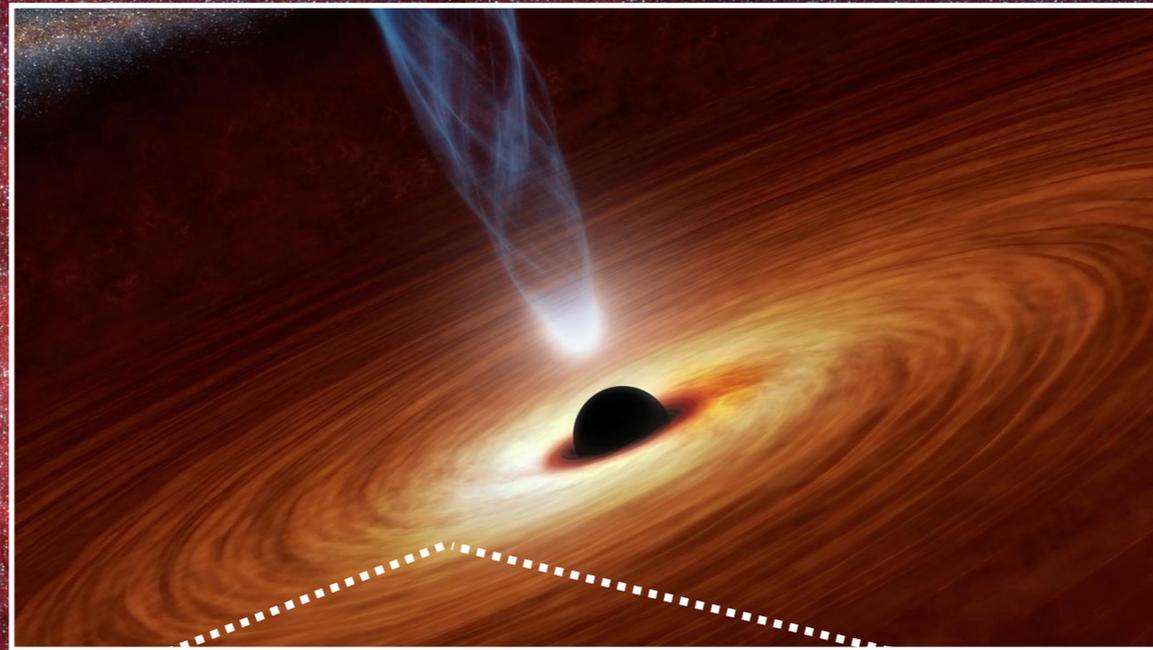# Electron Heating in Quasi-Relativistic Magnetic Reconnection

Michael Rowan
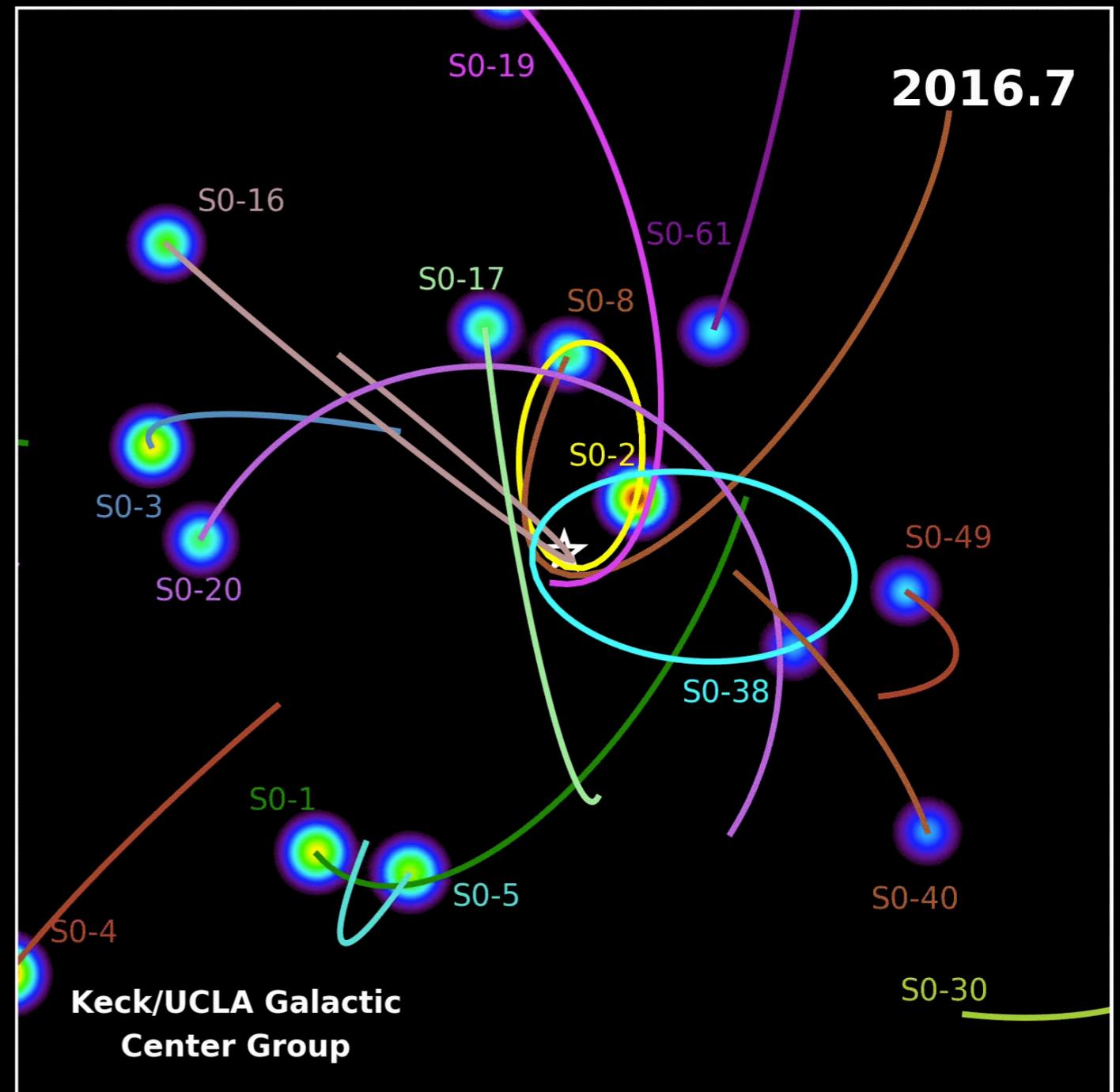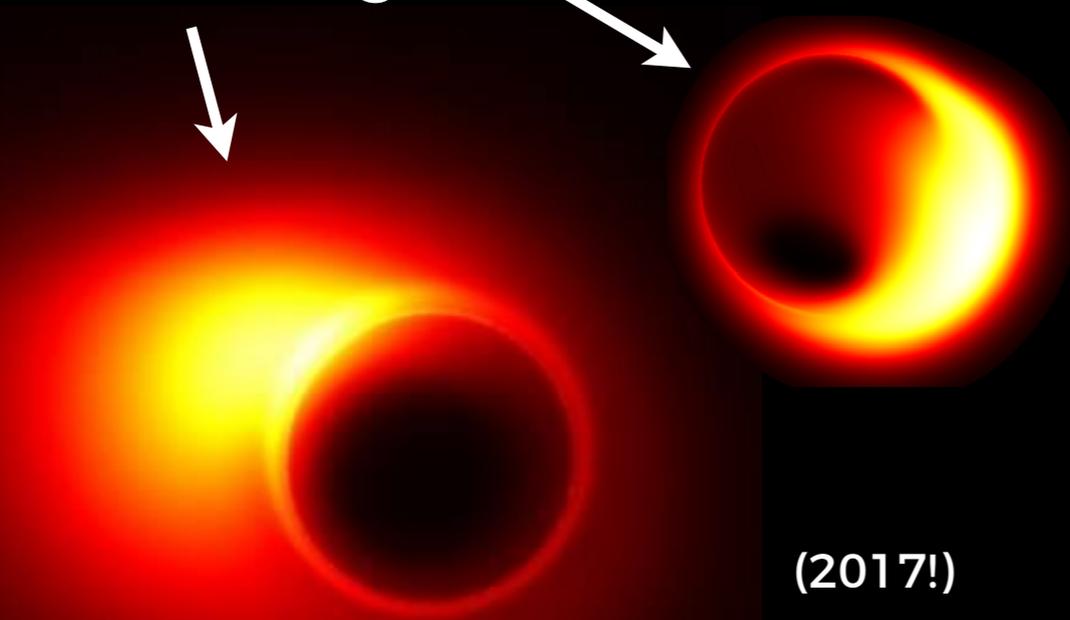Collaborators: Ramesh Narayan, Lorenzo Sironi
12/16/2016

NASA/JPL-Caltech/S. Stolovy (SSC/Caltech)

The importance of black holes

- Highly energetic
  - Quasars
  - GRBs
- Feedback into environment
  - Galaxy evolution
  - Large-scale structure
- EHT
  - Image black holes
  - M87, Sgr A*

(2017!)
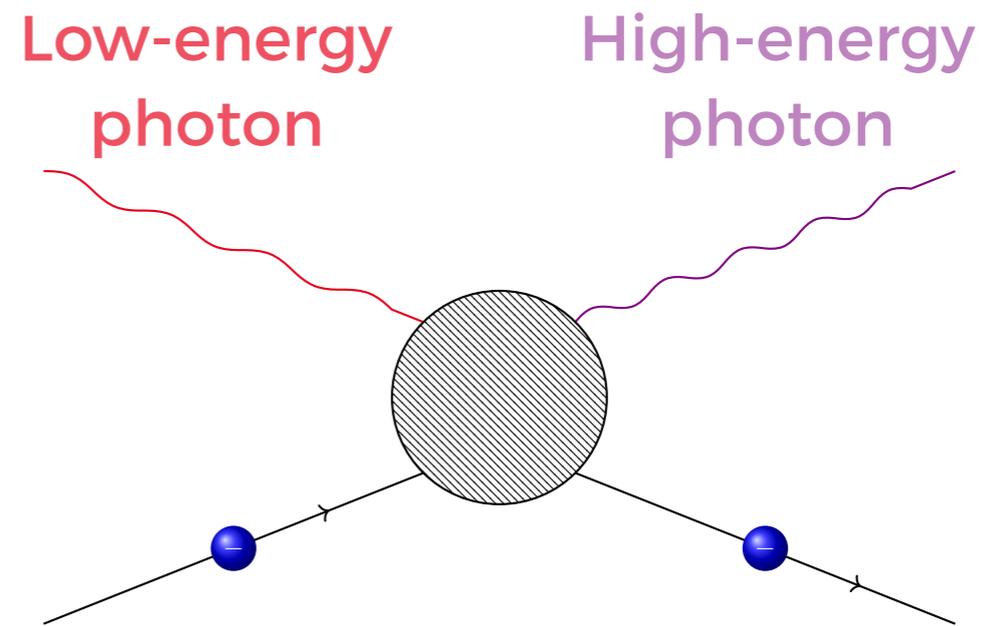
Credit: A. E. Broderick

2016.7

S0-19
S0-16
S0-61
S0-17
S0-8
S0-3
S0-2
S0-49
S0-20
S0-38
S0-1
S0-5
S0-40
S0-4
S0-30

Keck/UCLA Galactic Center Group

Electrons in accretion disk make radiation

Energetic electrons can produce radiation through various processes:

Synchrotron

Low-energy photons
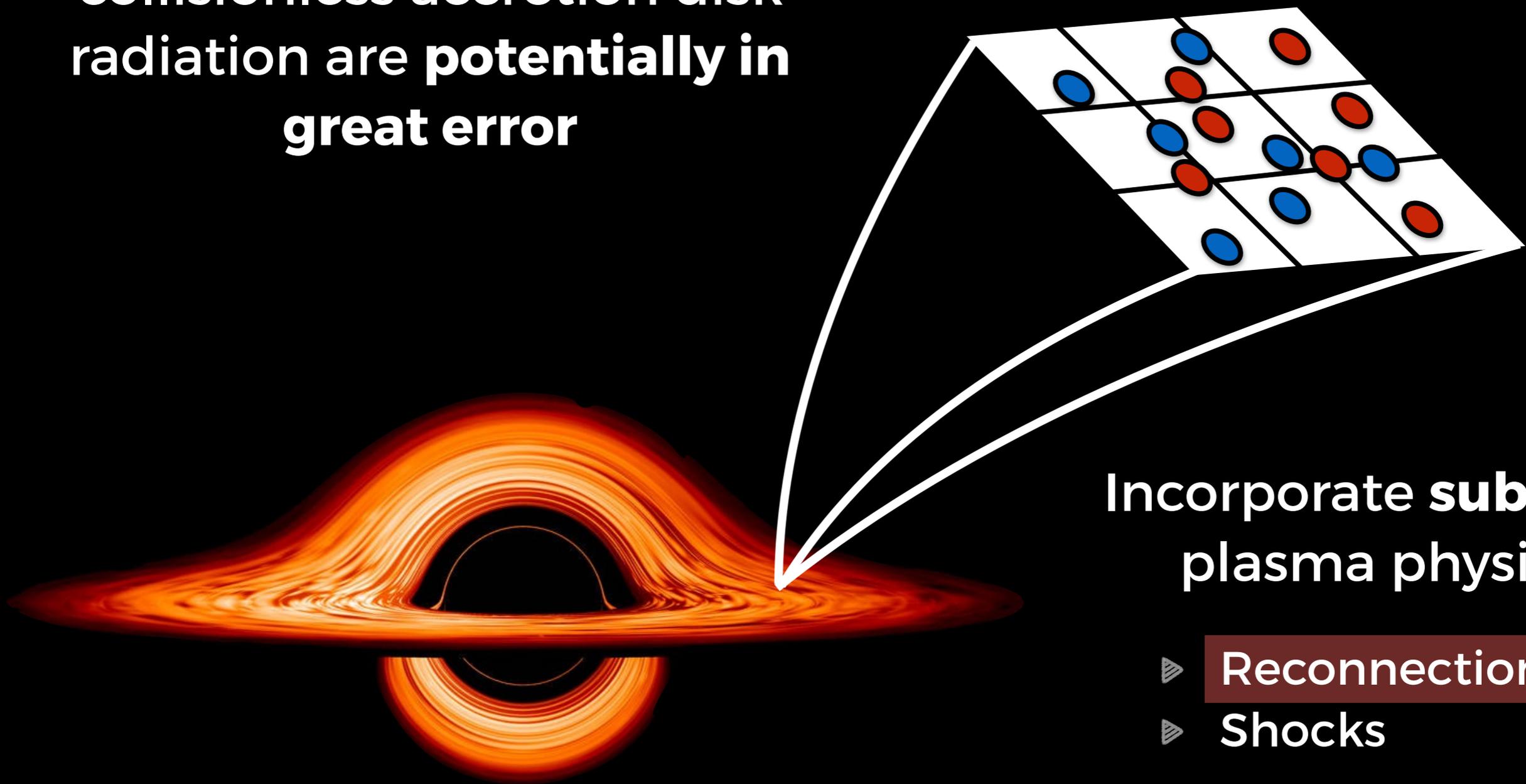
B-field

Inverse Compton

Low-energy photon
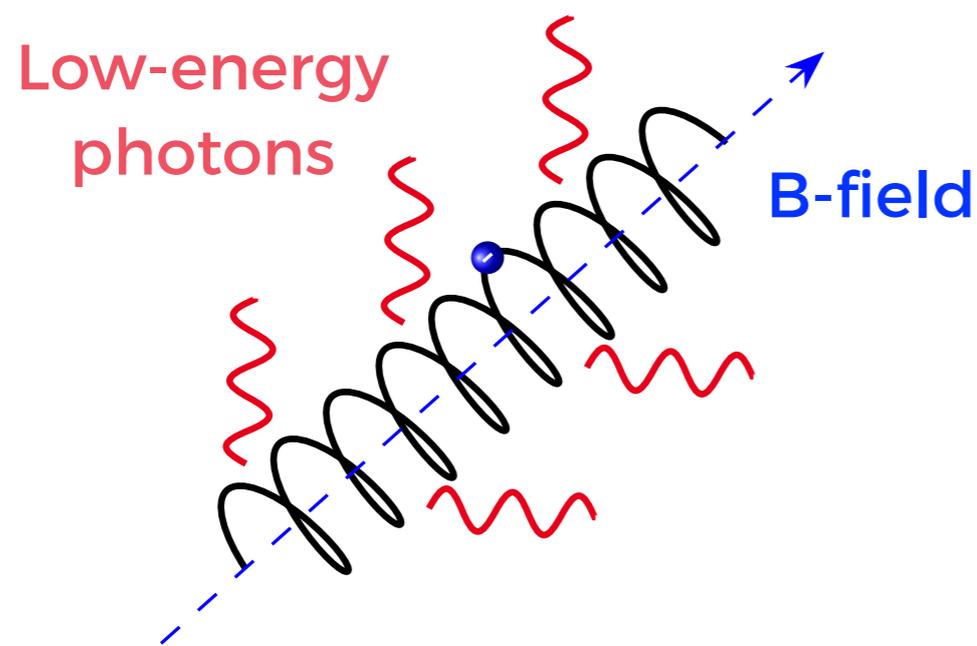
High-energy photon

Electrons are responsible for observed radiation profiles, yet the production of energetic electrons is **unresolved** in global simulations

Reconnection can energize particles

▷ Rearrangement of magnetic field lines

▷ Magnetic energy → particle kinetic energy
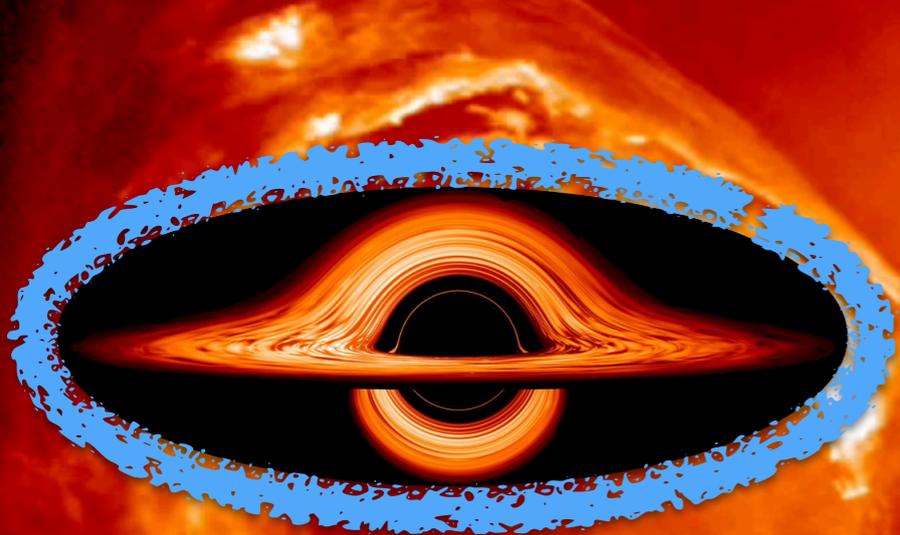
NASA SDO
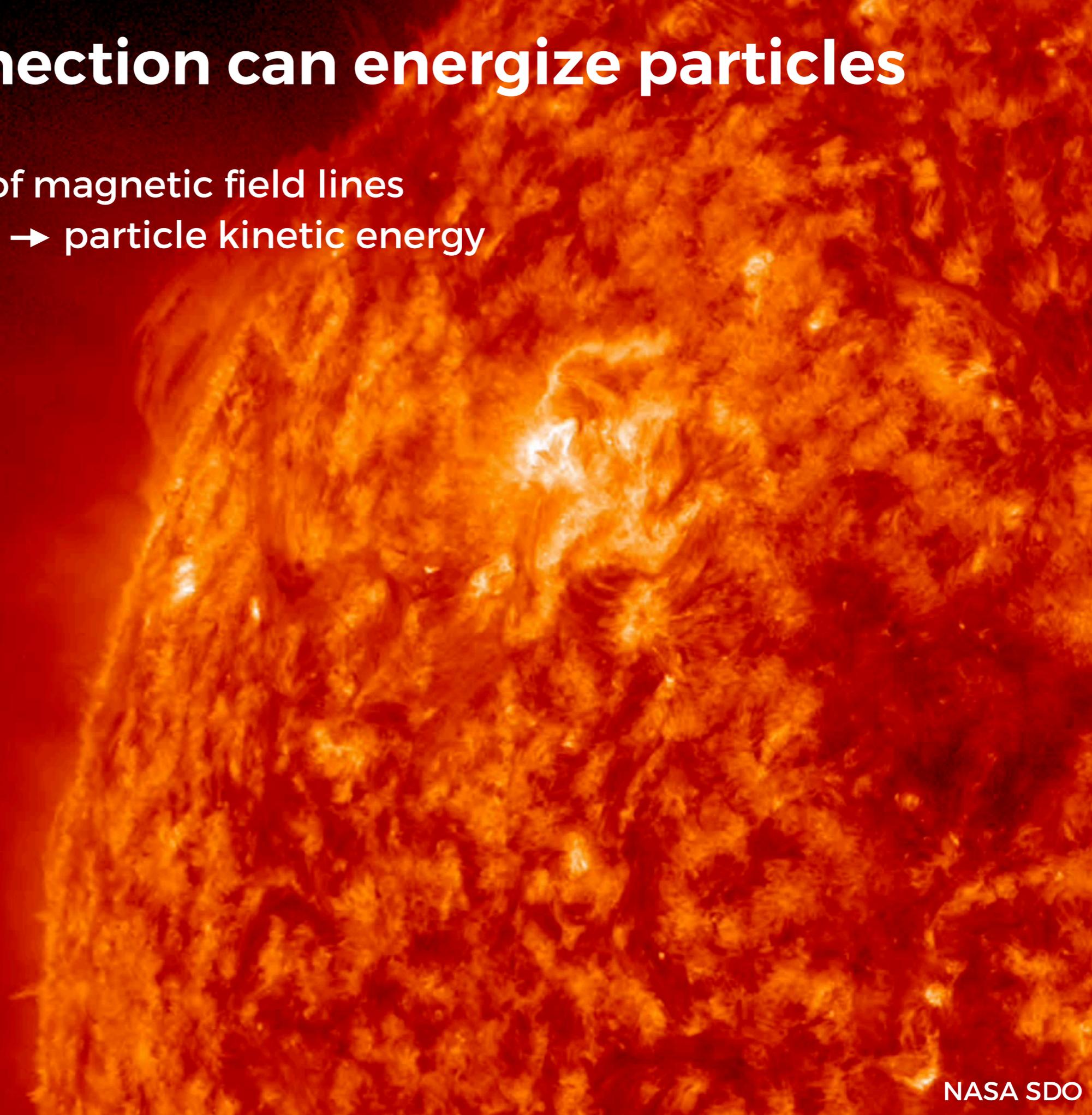
Reconnection can energize particles

▷ Rearrangement of magnetic field lines
▷ Magnetic energy ➡ particle kinetic energy

Happens many places:
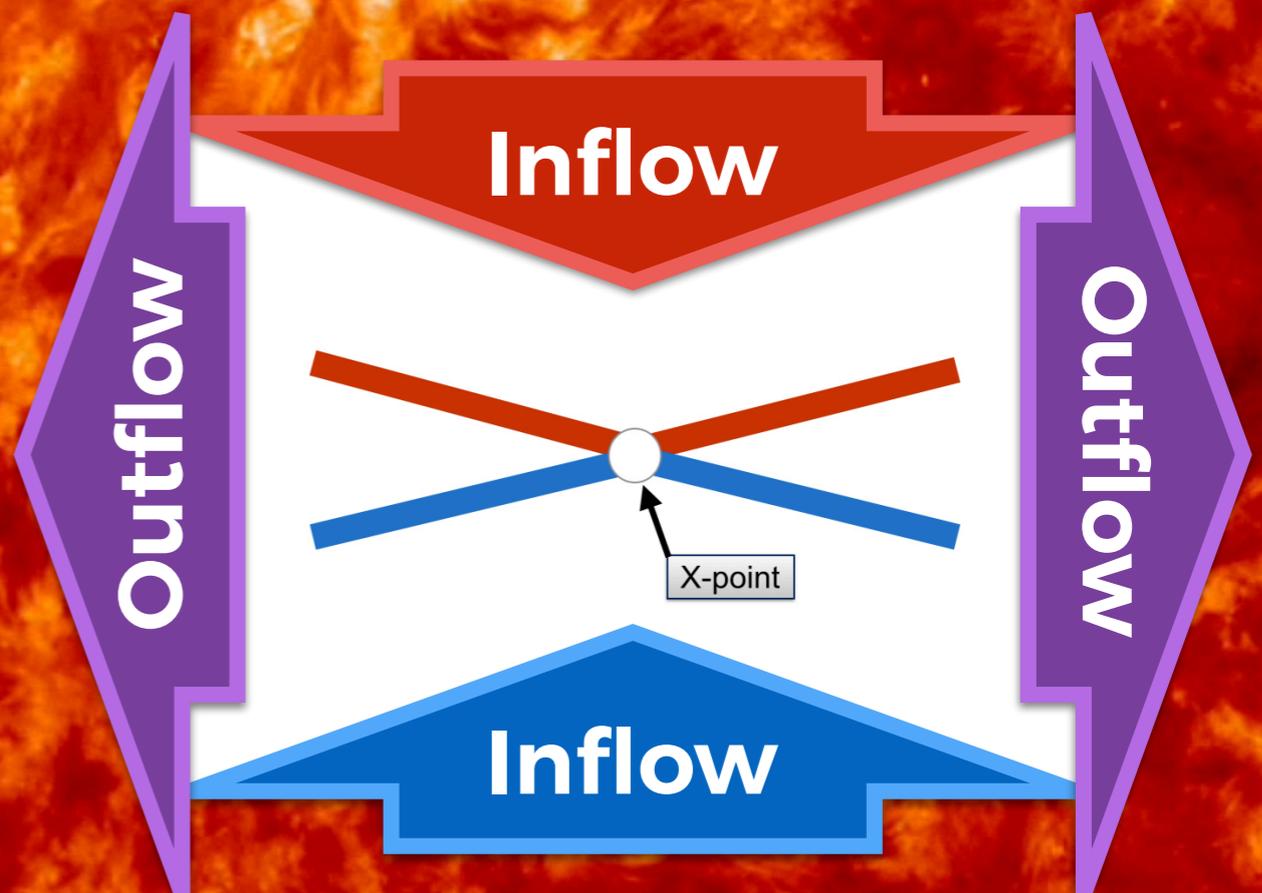 Chromosphere
  Magnetosphere
  Black hole coronae

NASA GSFC/ J. Schnittman

Inflow

Outflow

Outflow

X-point

Inflow

NASA SDO

Parameters: physical and computational

beta (of the ions)

$$\beta_i = \frac{n_i k_B T_i}{B^2/(8\pi)} = \frac{\text{thermal pressure}}{\text{magnetic pressure}}$$

sigma (of the ions)

$$\sigma_i = \frac{B^2/(4\pi)}{n_i m_i c^2} = \frac{\text{magnetic pressure} \, (\times 2)}{\text{rest-mass energy density}}$$

temperature ratio

$$\frac{T_e}{T_i} = \frac{\text{electron temperature}}{\text{ion temperature}}$$

Computational

d_{stripe}

dv_{stripe}

n_{stripe}

m_y

n_{times}

m_i/m_e

ppc

c/ω_{pe}

Full relativistic definition of sigma includes enthalpy

sigma, including enthalpy

$$\sigma_w = \frac{\frac{m_i}{m_e} + 1}{\frac{m_i}{m_e}\left(1 + \frac{\hat{\gamma}_i}{\hat{\gamma}_i - 1}\frac{k_B T_i}{m_i c^2}\right) + \left(1 + \frac{\hat{\gamma}_e}{\hat{\gamma}_e - 1}\frac{k_B T_e}{m_e c^2}\right)} \frac{B^2}{4\pi(n_i m_i + n_e m_e)c^2}$$

≈ 1 for a cold plasma

$\approx \frac{B^2}{4\pi n_i m_i c^2} \equiv \sigma_i$

For a high-beta (thermally 'hot') plasma, the contribution from the thermal pressure is non-negligible

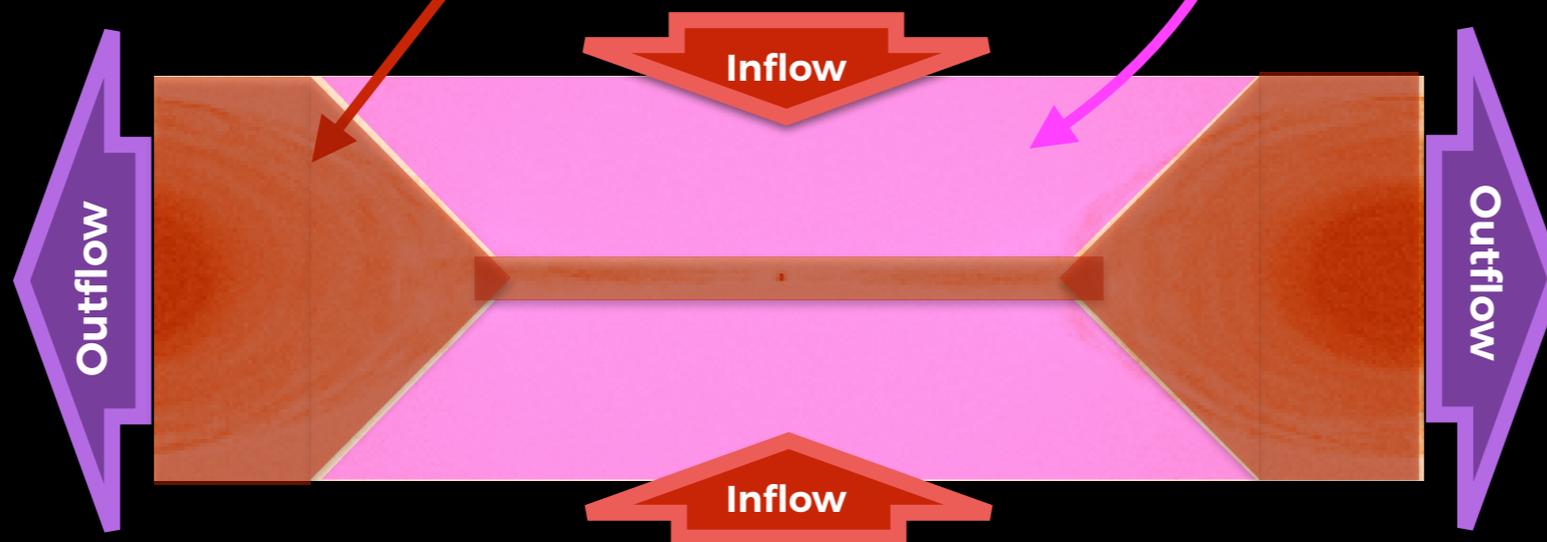One more important definition: Alfvén velocity, which describes the speed of magnetic waves

Alfvén velocity

$$\frac{v_A}{c} = \sqrt{\frac{\sigma_w}{1 + \sigma_w}}$$

A useful number we can extract from each simulation is the following dimensionless ratio:

$$M_{Te} = \frac{k_B T_{e,out} - k_B T_{e,in}}{B^2/(4\pi n)}$$

Inflow

Outflow

Outflow

Inflow

This is the ratio of increase in temperature to magnetic energy available for dissipation. It can be thought of as the 'efficiency' of reconnection.

How much are electrons heated during reconnection?

PIC simulations and observations of magnetic reconnection suggest that a constant fraction of inflowing magnetic energy is given to electrons

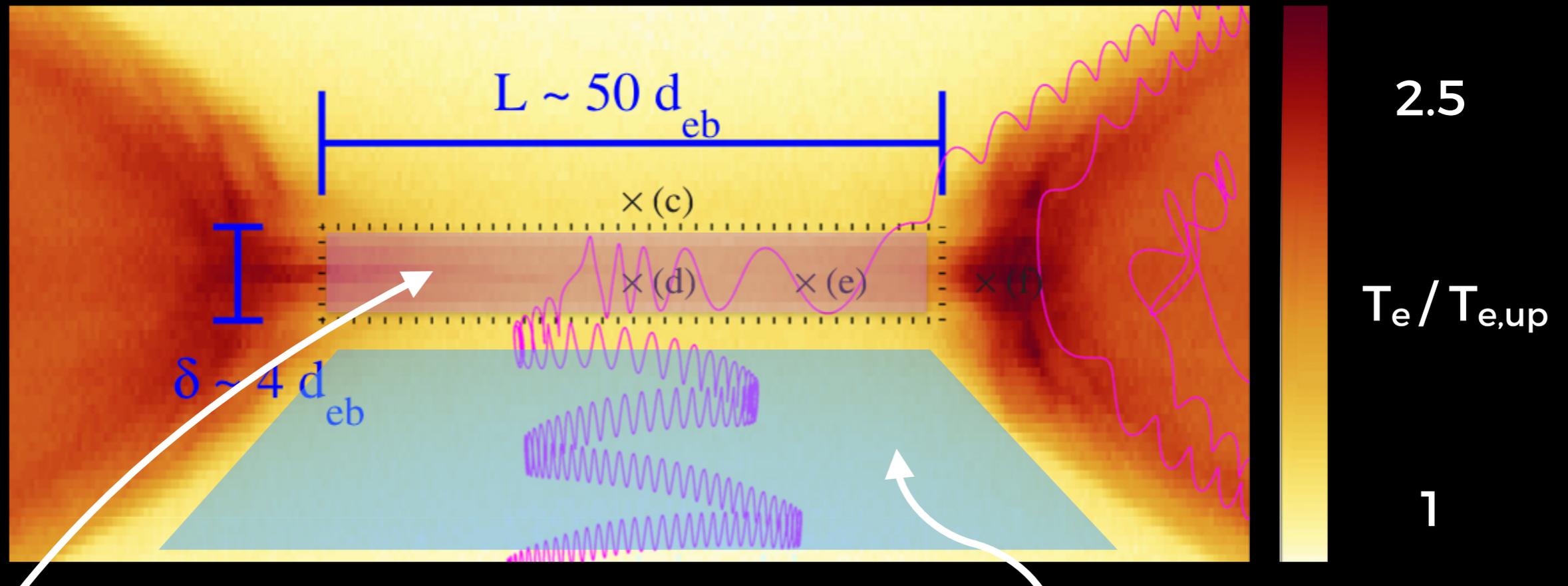$$M_{Te} = \frac{k_B T_{e,out} - k_B T_{e,in}}{m_i v_A^2}$$
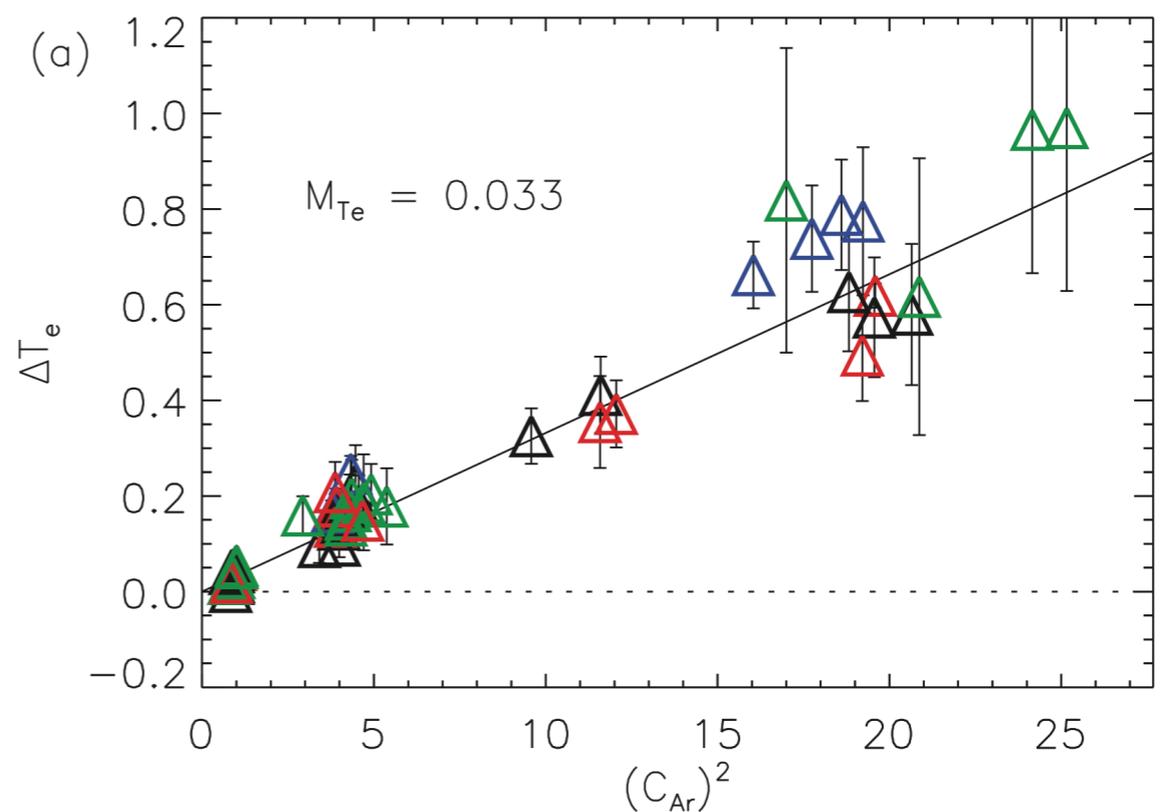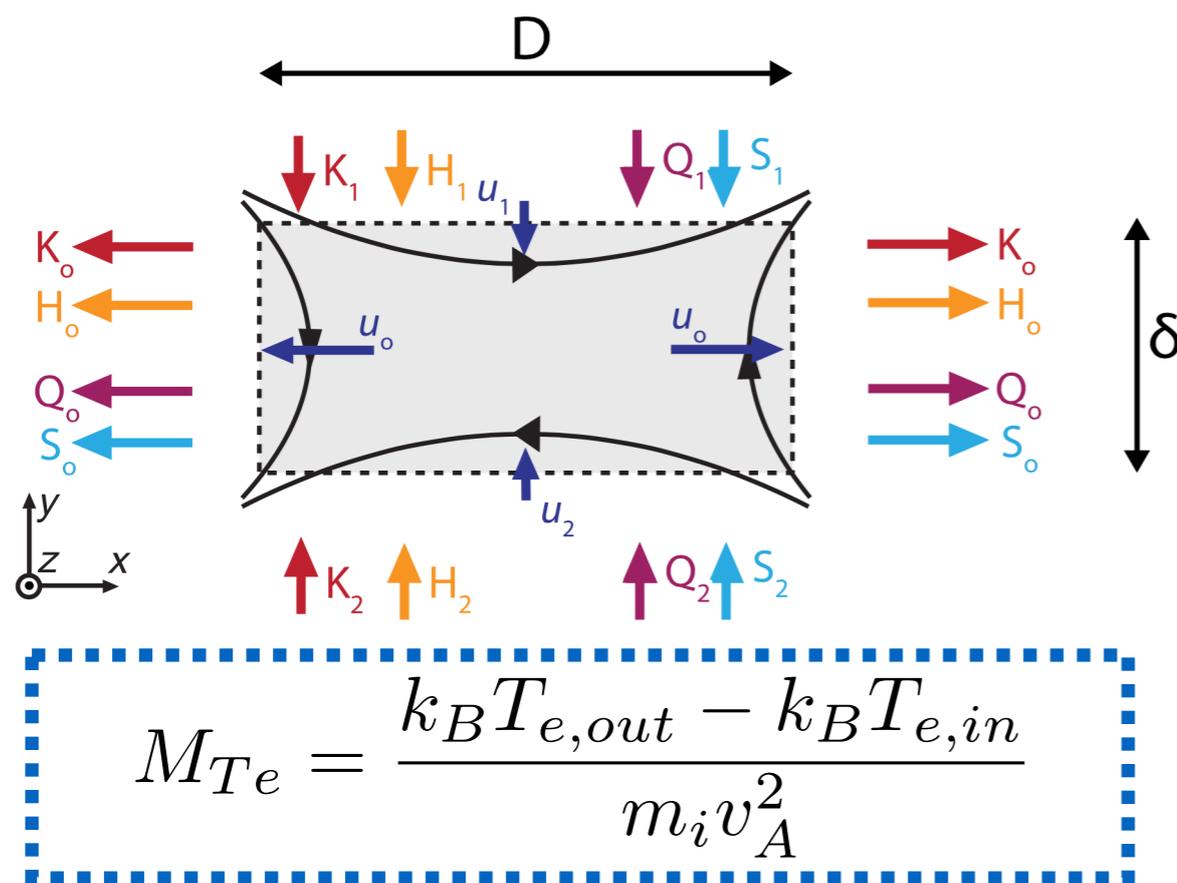
(Drake et al., 2014)

This fraction M_{Te} is remarkably independent of plasma parameters in the inflowing region
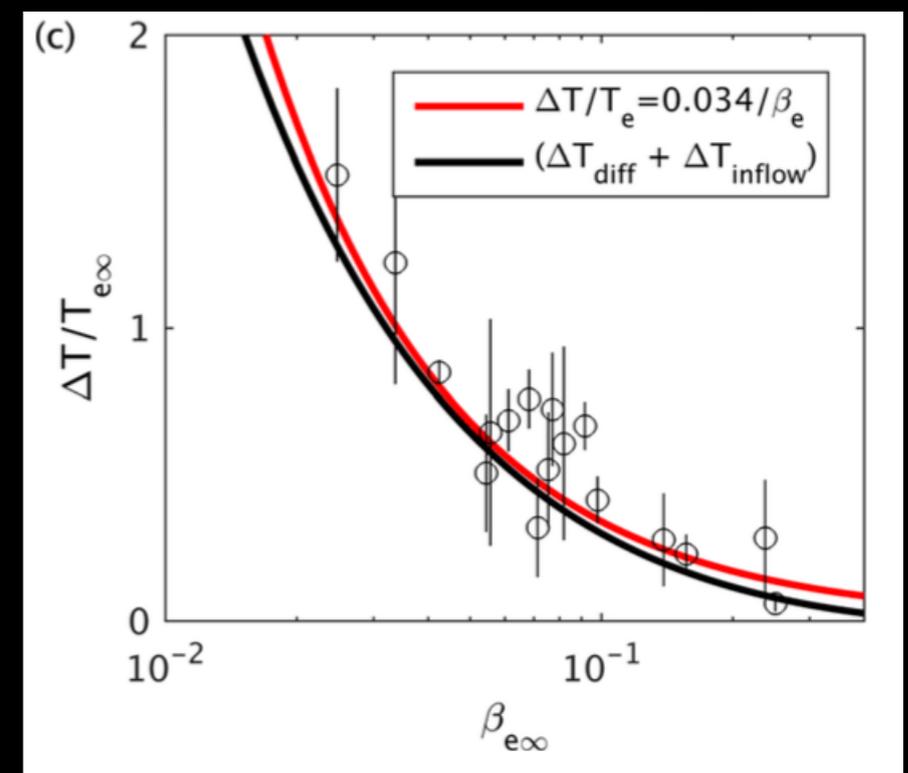
A model for the heating mechanism exists

(Le et al., 2016)

$$\frac{\Delta T_{e,tot}}{T_{e,up}} = \frac{\Delta T_{e,in}}{T_{e,up}} + \frac{\Delta T_{e,diff}}{T_{e,up}} \simeq \frac{0.034}{\beta_{e,up}}$$

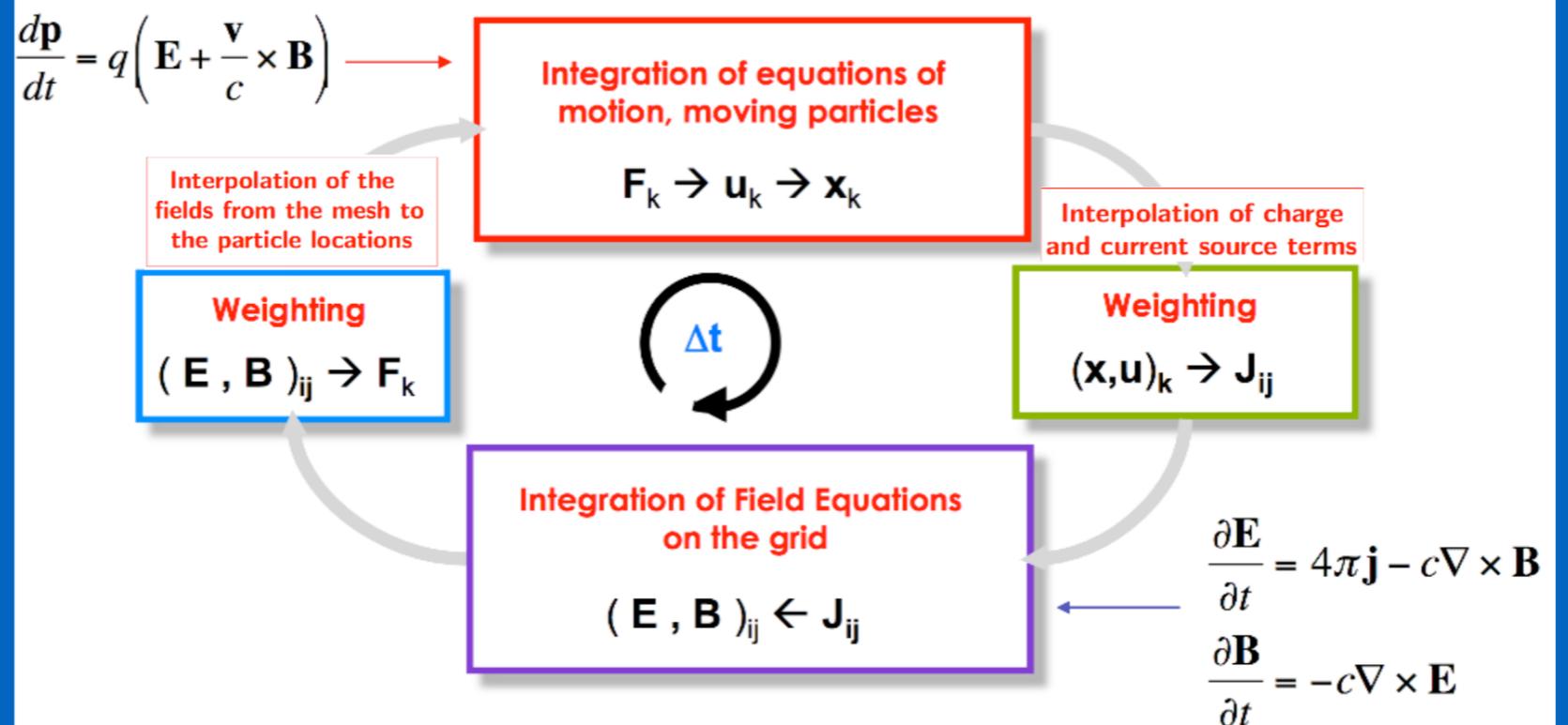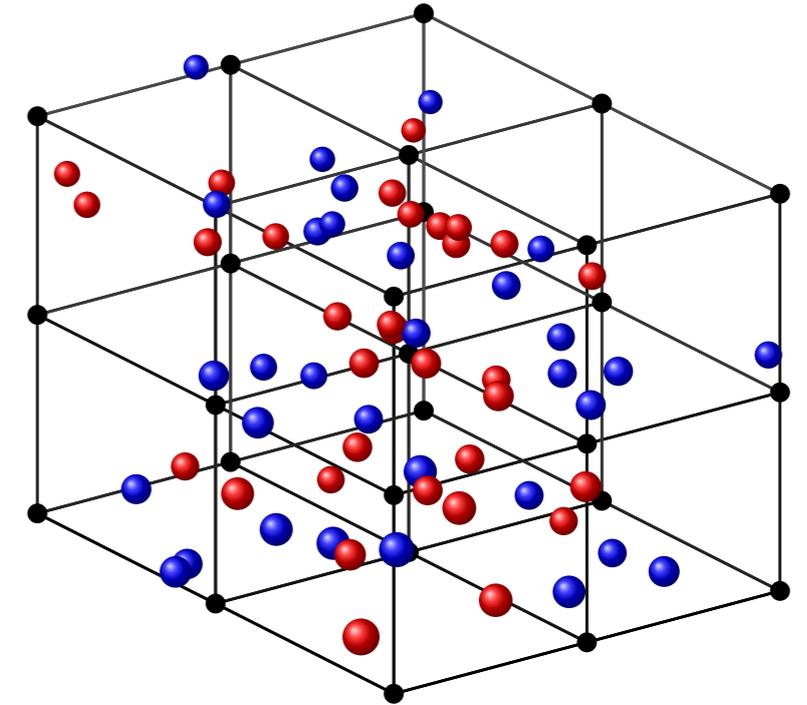**The model (middle terms) agrees with the empirical scaling (last term)**

(Le et al., 2016)

The quasi-relativistic regime is relatively unexplored

Parameters

σ_w	β_i	T_e/T_i	$\Delta\gamma_i$
0.1	0.0078125	0.1	0.000406687
0.1	0.0078125	0.3	0.000406767
0.1	0.0078125	1	0.000407051
0.1	0.03125	0.1	0.00163203
0.1	0.03125	0.3	0.00163334
0.1	0.03125	1	0.00163818
0.1	0.125	0.1	0.00661497
0.1	0.125	0.3	0.00663803
0.1	0.125	1	0.00673223
0.1	0.5	0.1	0.0280133
0.1	0.5	0.3	0.0285164
0.1	0.5	1	0.0308345
0.1	2.	0.1	0.155222
0.1	2.	0.3	0.178254
0.1	2.	1	0.394336
0.3	0.0078125	0.1	0.0012227
0.3	0.0078125	0.3	0.00122343
0.3	0.0078125	1	0.0012261
0.3	0.03125	0.1	0.00493921
0.3	0.03125	0.3	0.00495179
0.3	0.03125	1	0.00500182
0.3	0.125	0.1	0.0205981
0.3	0.125	0.3	0.0208554
0.3	0.125	1	0.022019
0.3	0.5	0.1	0.102084
0.3	0.5	0.3	0.110952
0.3	0.5	1	0.163062

Use **PiC simulation**. Choose parameters so that inflow/ outflow electrons are **moderately** relativistic

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)$$

Integration of equations of motion, moving particles

$$\mathbf{F}_k \rightarrow \mathbf{u}_k \rightarrow \mathbf{x}_k$$

Interpolation of the fields from the mesh to the particle locations

Interpolation of charge and current source terms

Weighting

$$(\mathbf{E},\mathbf{B})_{ij} \rightarrow \mathbf{F}_k$$

Δt

Weighting

$$(\mathbf{x},\mathbf{u})_k \rightarrow \mathbf{J}_{ij}$$

Integration of Field Equations on the grid

$$(\mathbf{E},\mathbf{B})_{ij} \leftarrow \mathbf{J}_{ij}$$

$$\frac{\partial\mathbf{E}}{\partial t} = 4\pi\mathbf{j} - c\nabla\times\mathbf{B}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}$$

if they are at about the same position and share about the same velocity. Hence, we define $f(\boldsymbol{x}, \boldsymbol{v}, t)$ as the particle distribution function, which represents the number density of particles found near the point $(\boldsymbol{x}, \boldsymbol{v})$ in phase space. Specifically, the number of particles located within intervals d^3x about $\boldsymbol{x}$ and d^3v about $\boldsymbol{v}$ is given by

$$dN = f(\boldsymbol{x}, \boldsymbol{v}, t)d^3x\,d^3v \ . \tag{2}$$

$\boldsymbol{X} \equiv (\boldsymbol{x}, \boldsymbol{v})$ is phase space coordinates and $\dot{\boldsymbol{X}} = (\boldsymbol{v}, \boldsymbol{a})$

$$\dot{x}_i = v_i \ , \quad \dot{v}_i = a_i = \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_i \times \boldsymbol{B}}{c}\right)$$

$$\frac{\partial f}{\partial t} + \nabla \cdot (f\boldsymbol{v}) + \nabla_v \cdot (f\boldsymbol{a}) = 0$$

$$\frac{Df}{Dt} \equiv \left(\frac{\partial}{\partial t} + \dot{\boldsymbol{X}}\frac{\partial}{\partial \boldsymbol{X}}\right)f = 0$$

$Df/Dt = 0$ is known as *Liouville's theorem*. It states that the distribution function f is constant along particle trajectories in phase space (when $\nabla_v \cdot \boldsymbol{a} = 0$).

The Vlasov-Maxwell equations

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{v}} f = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\boldsymbol{j}\ ,$$

$$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}\ ,$$

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho\ ,$$

$$\nabla \cdot \boldsymbol{B} = 0\ .$$

$$\rho = \sum_{\text{species}} q \int f(\boldsymbol{x}, \boldsymbol{v}, t) d^3 v\ ,$$

$$\boldsymbol{j} = \sum_{\text{species}} q \int \boldsymbol{v} f(\boldsymbol{x}, \boldsymbol{v}, t) d^3 v\ .$$

Two options:

• Discretize the Vlasov equation on a grid in phase space:

1. computationally expensive to solve in 6+1 dimensions

2. how to determine the boundaries of the grid in momentum space?

3. what if $f<0$?

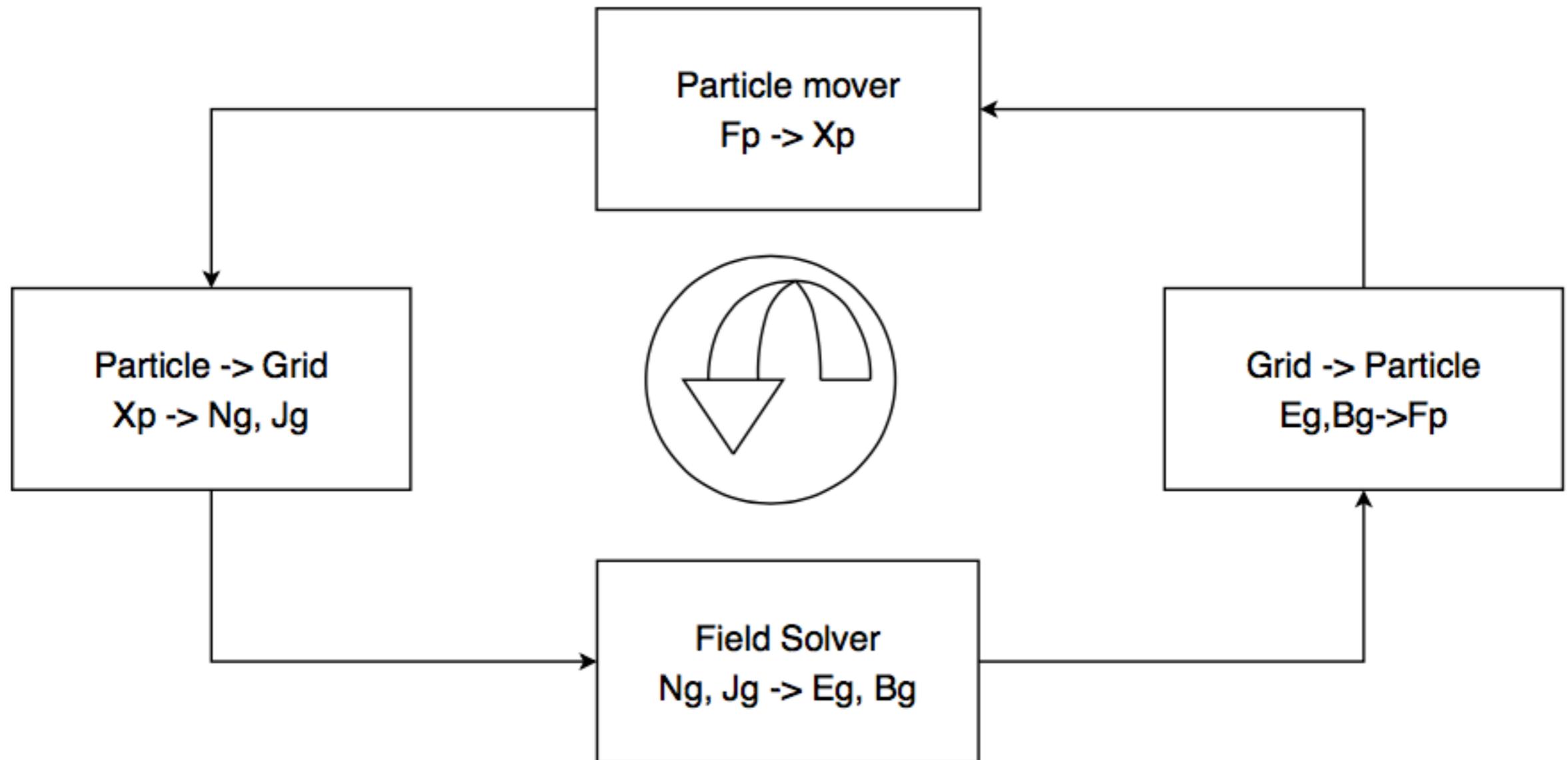• Sample the phase space density with particles, and follow them as LAGRANGIAN tracers.
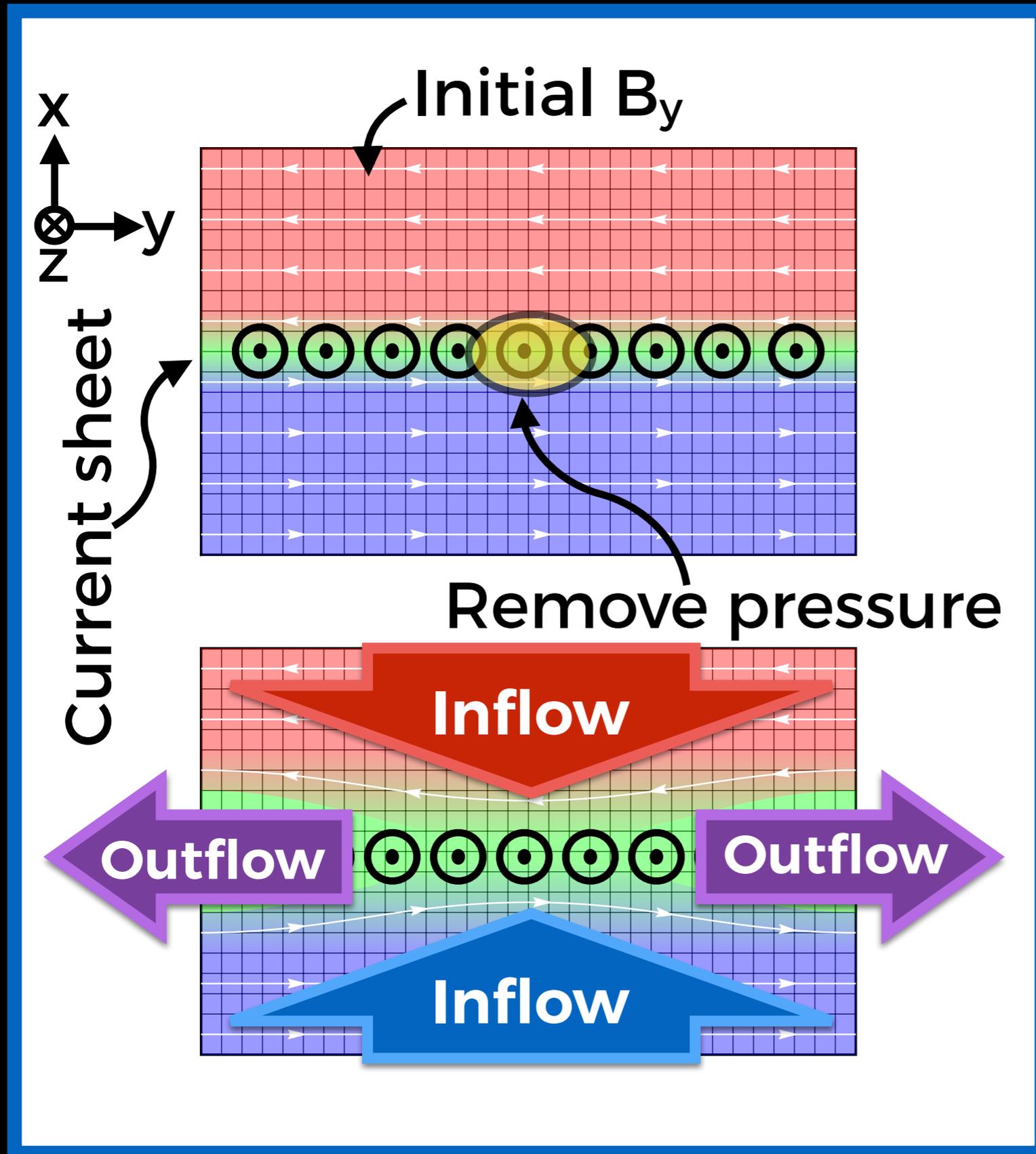

P — unoccupied phase space (wasted cells)

plasma distribution function — *X*
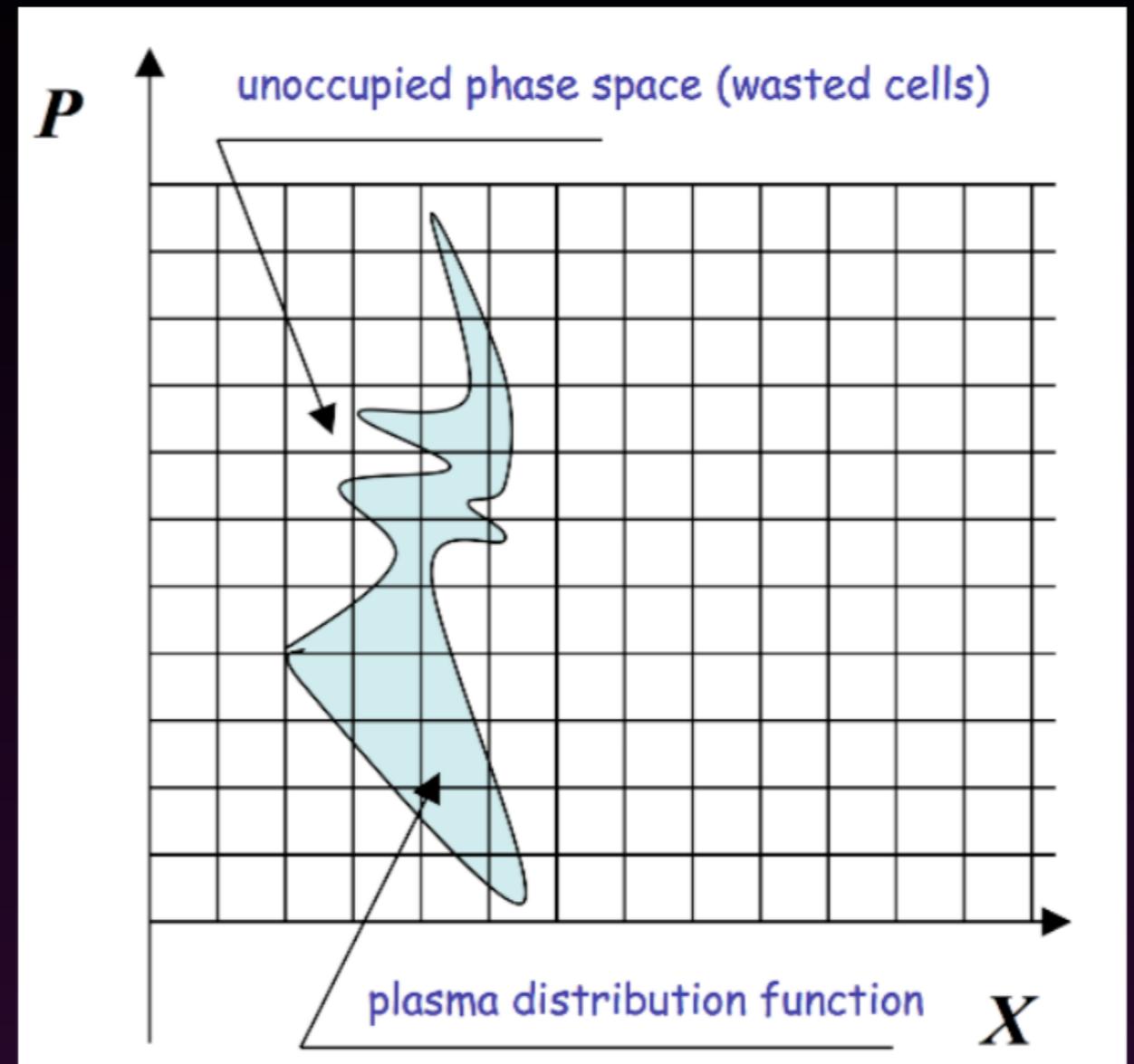
Macro-particles vs. real particles
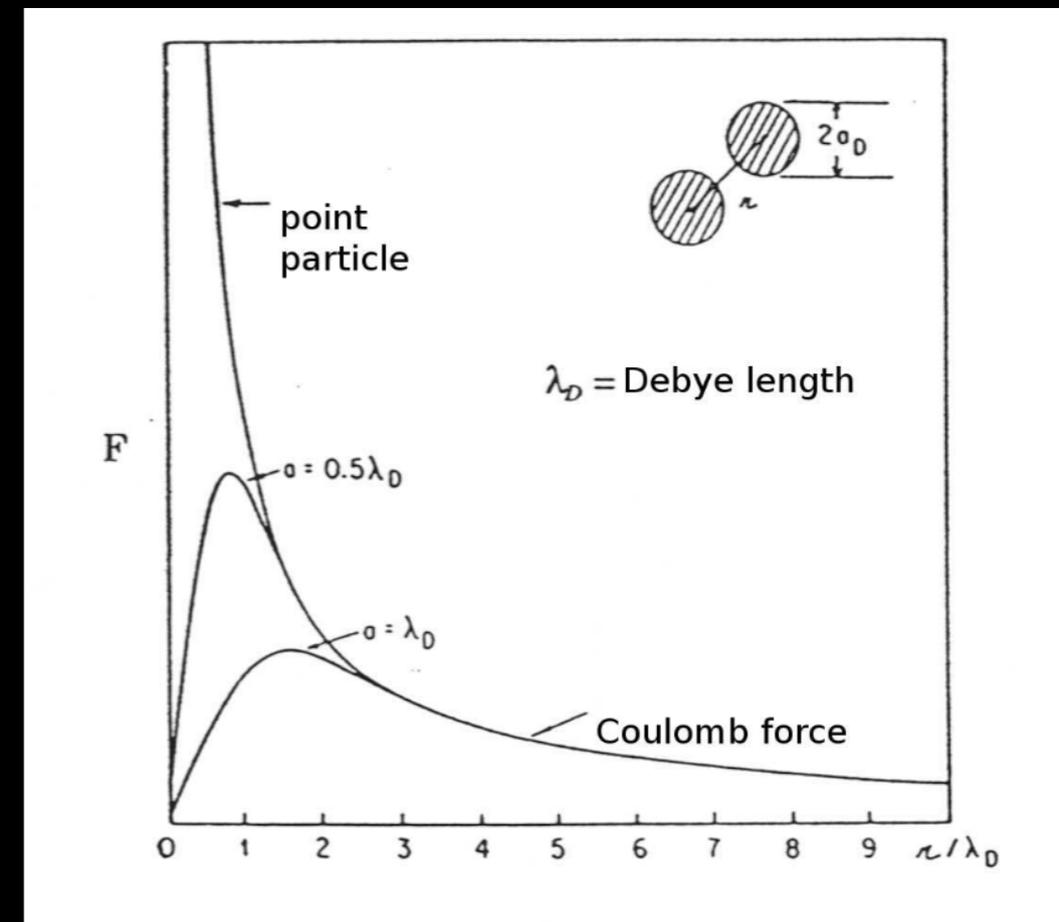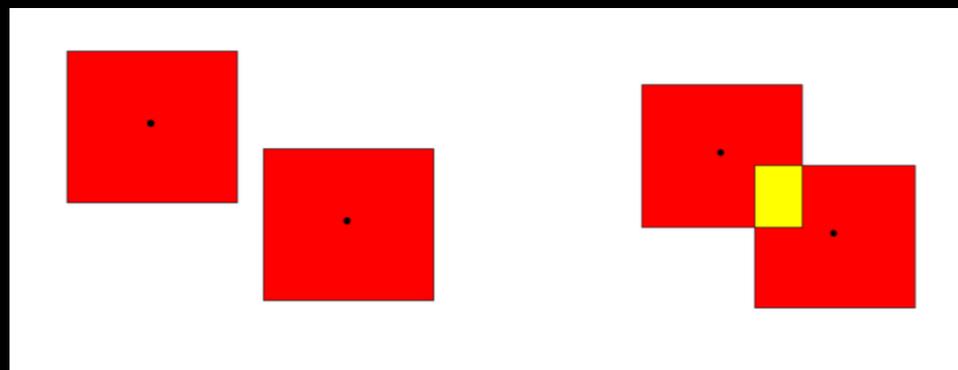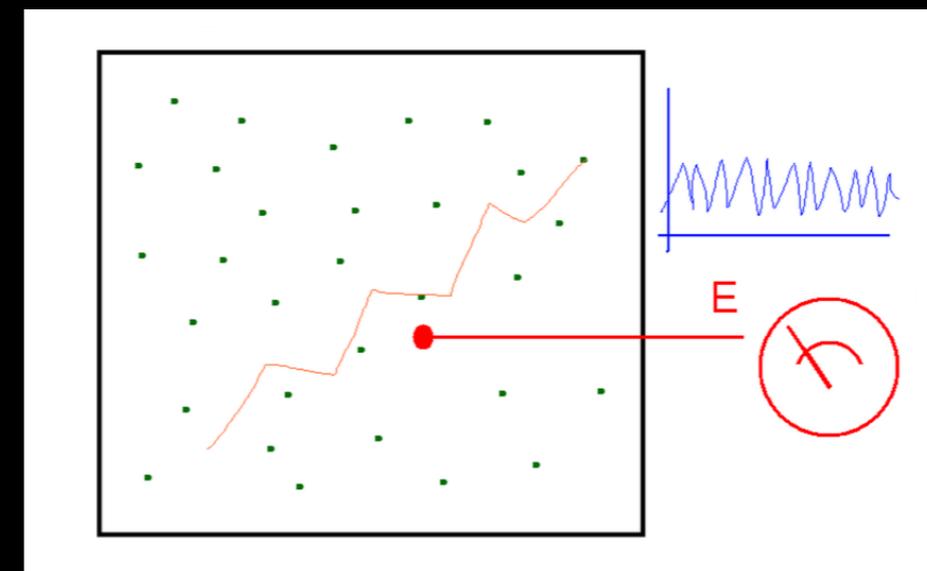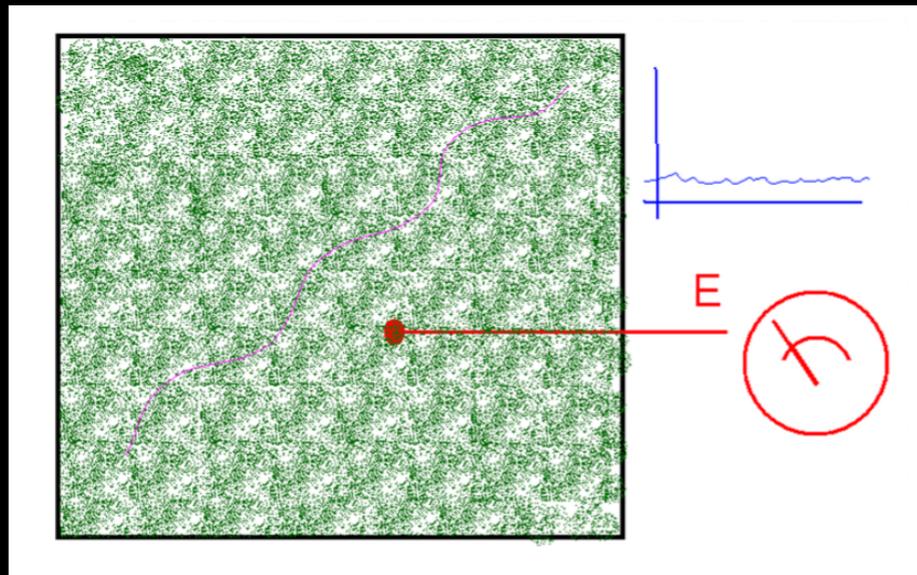
$$\Lambda = \frac{E_{th}}{E_{pot}} = \frac{4\pi\epsilon_0 a k T}{q^2}$$

Particle-in-cell loop

Start with alternating $\vec{B}$-field and trigger reconnection

B-field initialized in Harris equilibrium
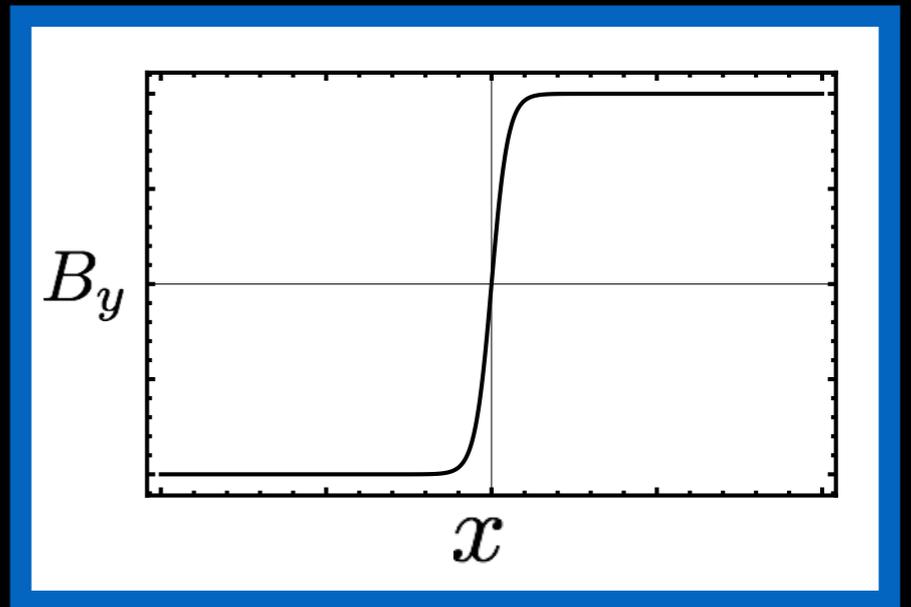
$$\mathbf{B} = B_0 \tanh (x/L)\mathbf{e}_y$$

- Hot, overdense strip of particles at beginning (green)
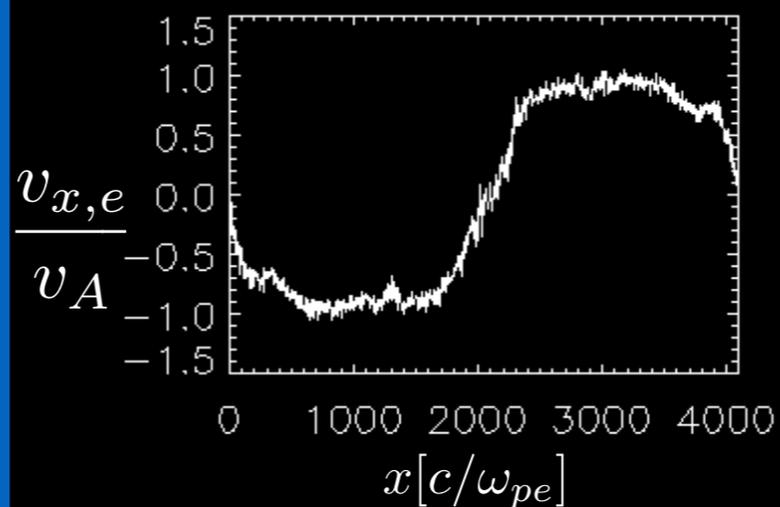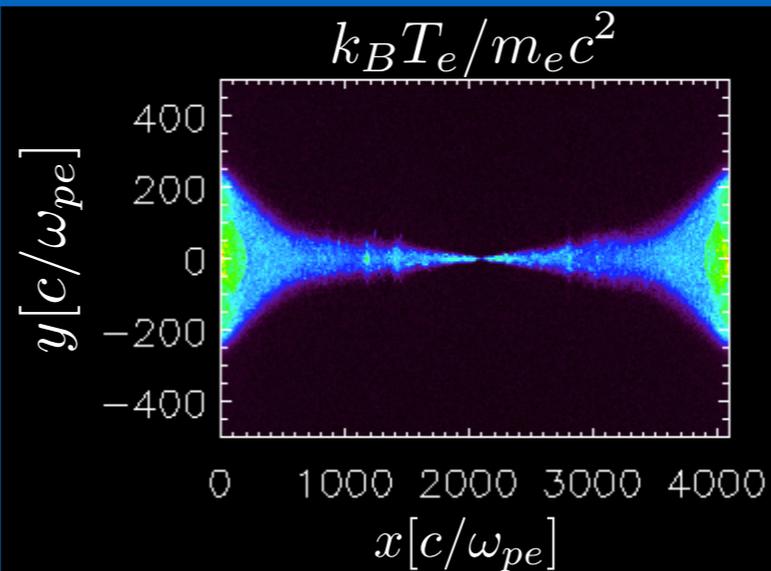- Remove the particle pressure in center to drive reconnection

Boundary conditions

| Outflow	Periodic	Adaptive
▷ Particles escape along x-dir. ▷ Allows for study of long-term evolution of system	▷ No particles are lost ▷ However, sensitive to boundaries after ½ Alfvén crossing-time	▷ Modified version of outflow boundary condition ▷ Includes additional controls necessary for high-beta case
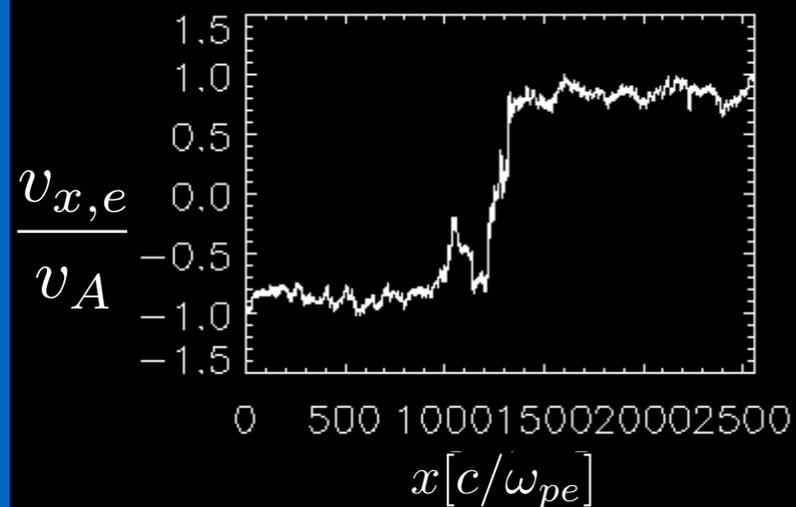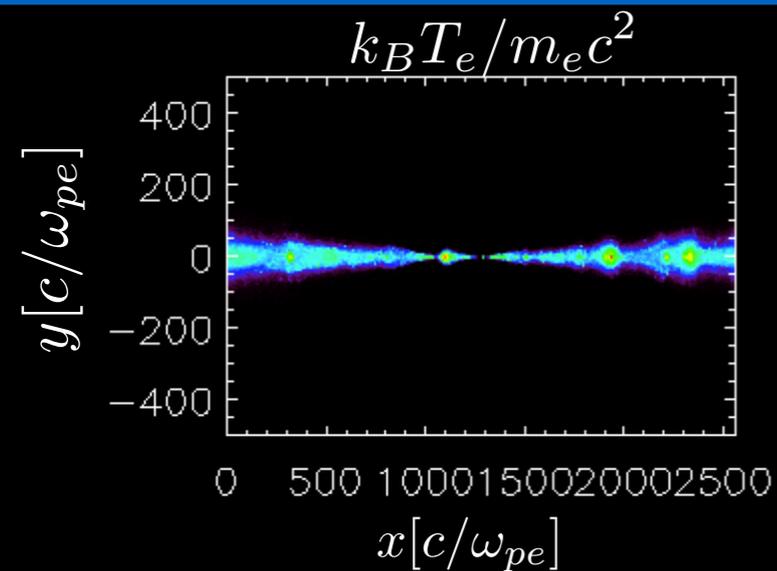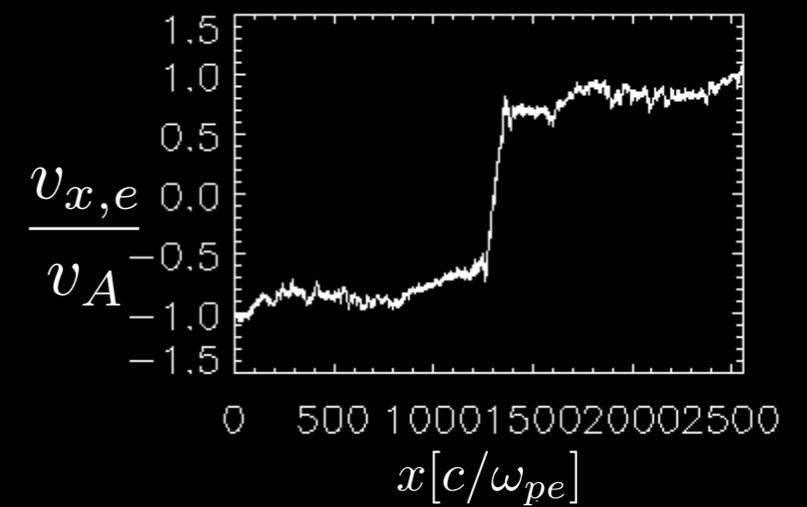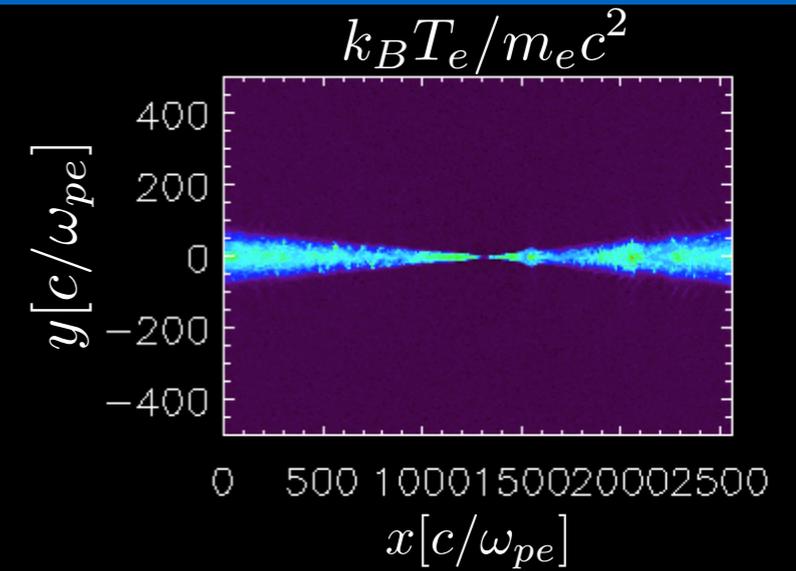
The plasma reaches a quasi-steady state

- To extract a meaningful outflow temperature, temperature profile should be flat
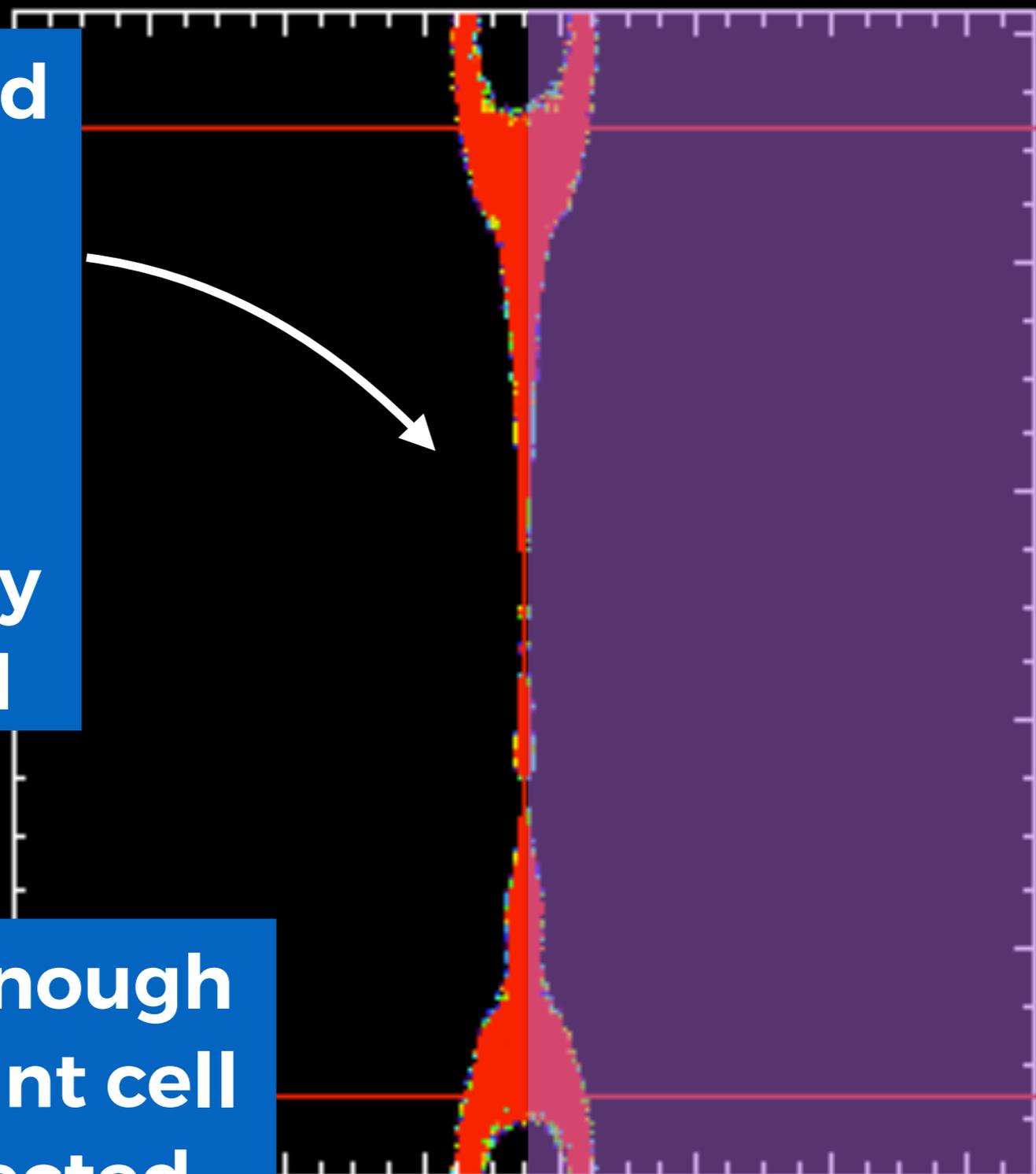- Alfvén velocity should be saturated in current sheet

Alfvén velocity

$$\frac{v_A}{c} = \sqrt{\frac{\sigma_w}{1 + \sigma_w}}$$
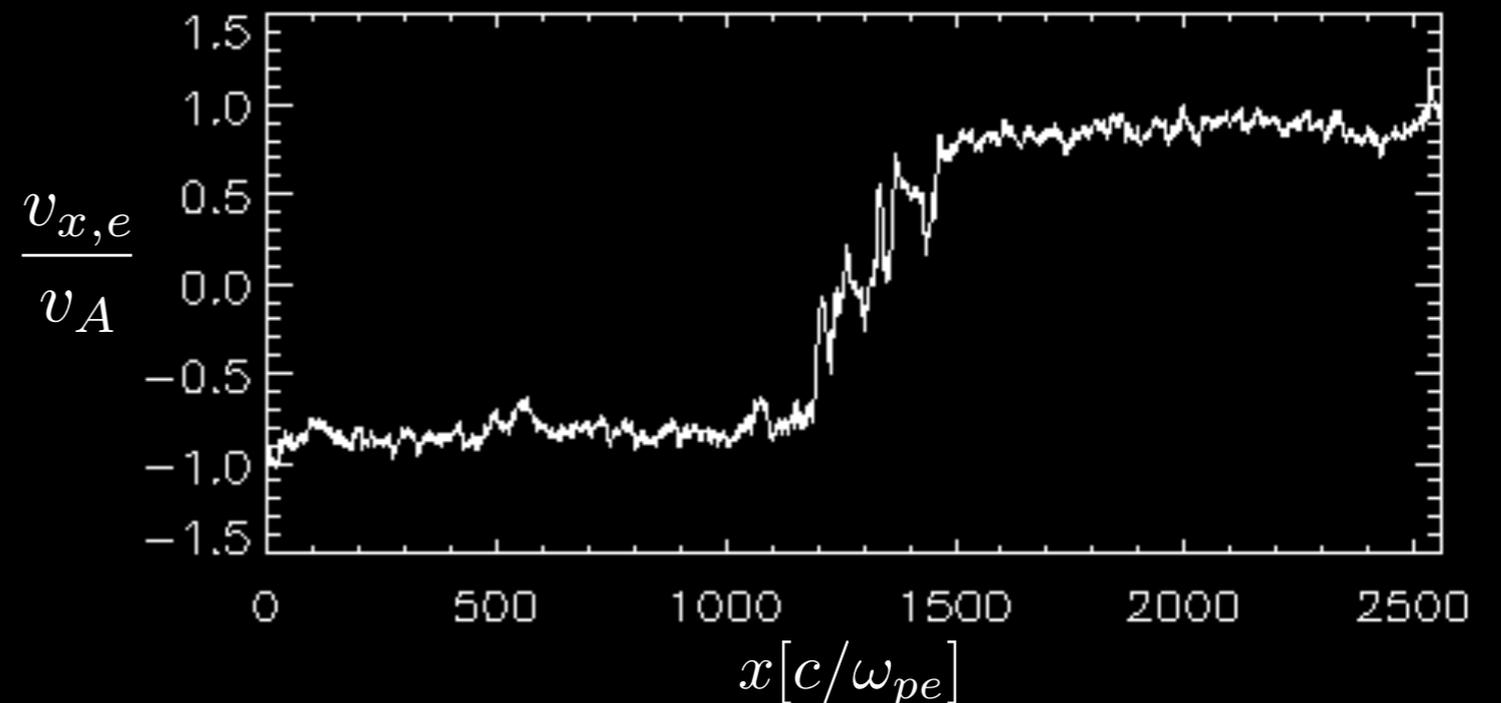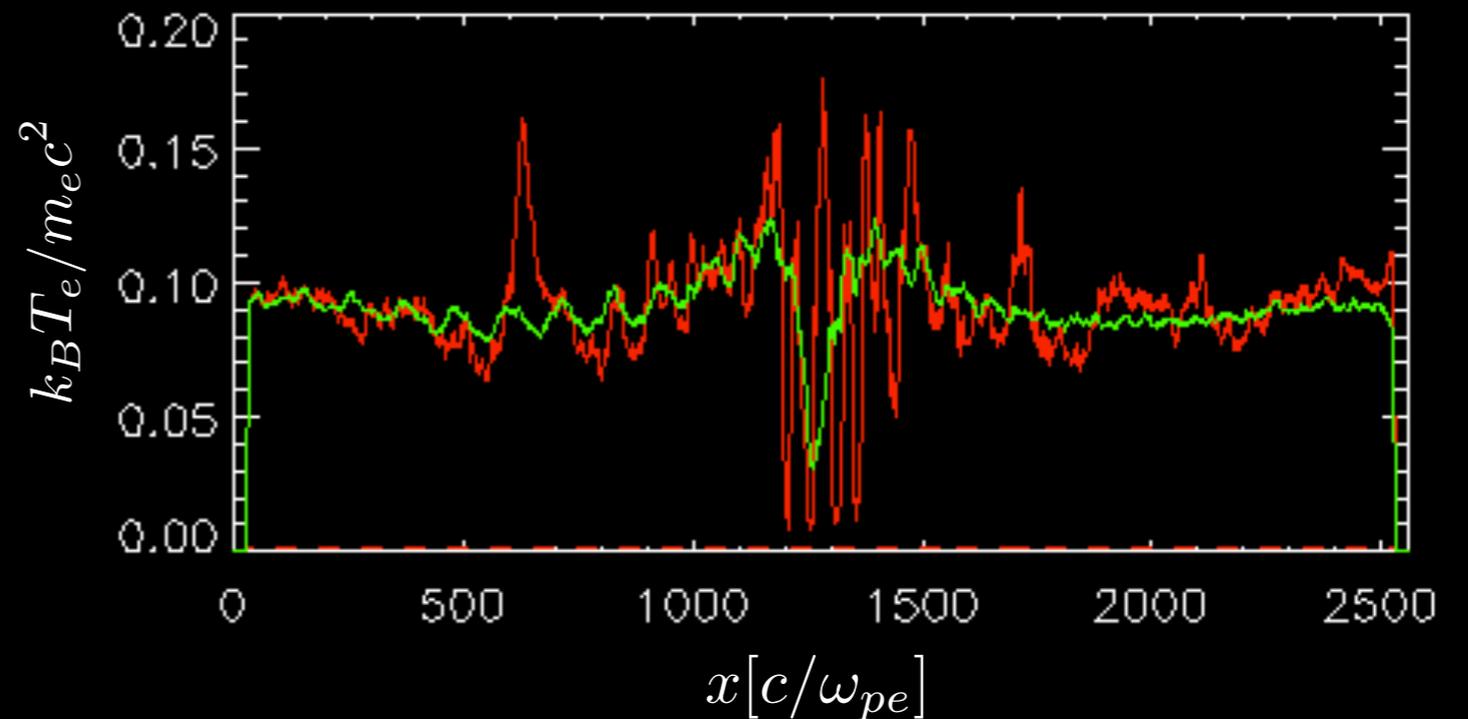
How to identify where reconnection has happened?

Track tagged particles; measure ratio of tagged to total density in each cell

Tag particles on right

Region selected is insensitive to the particular threshold value

If there is enough mixing, count cell as reconnected

This is a low-beta plasma...

▷ The circular substructures are 'magnetic islands'
▷ They form due to the tearing instability
▷ Islands heat particles by 'bouncing'

$$k_b T_e / m_e c^2$$

Plot annotations:
$m_i/m_e = 25$
$\beta_i = 0.008$
$\sigma_i = 0.1$
$T_{e,in}/T_{i,in} = 0.1$

x-axis: $x, [c/\omega_{pe}]$
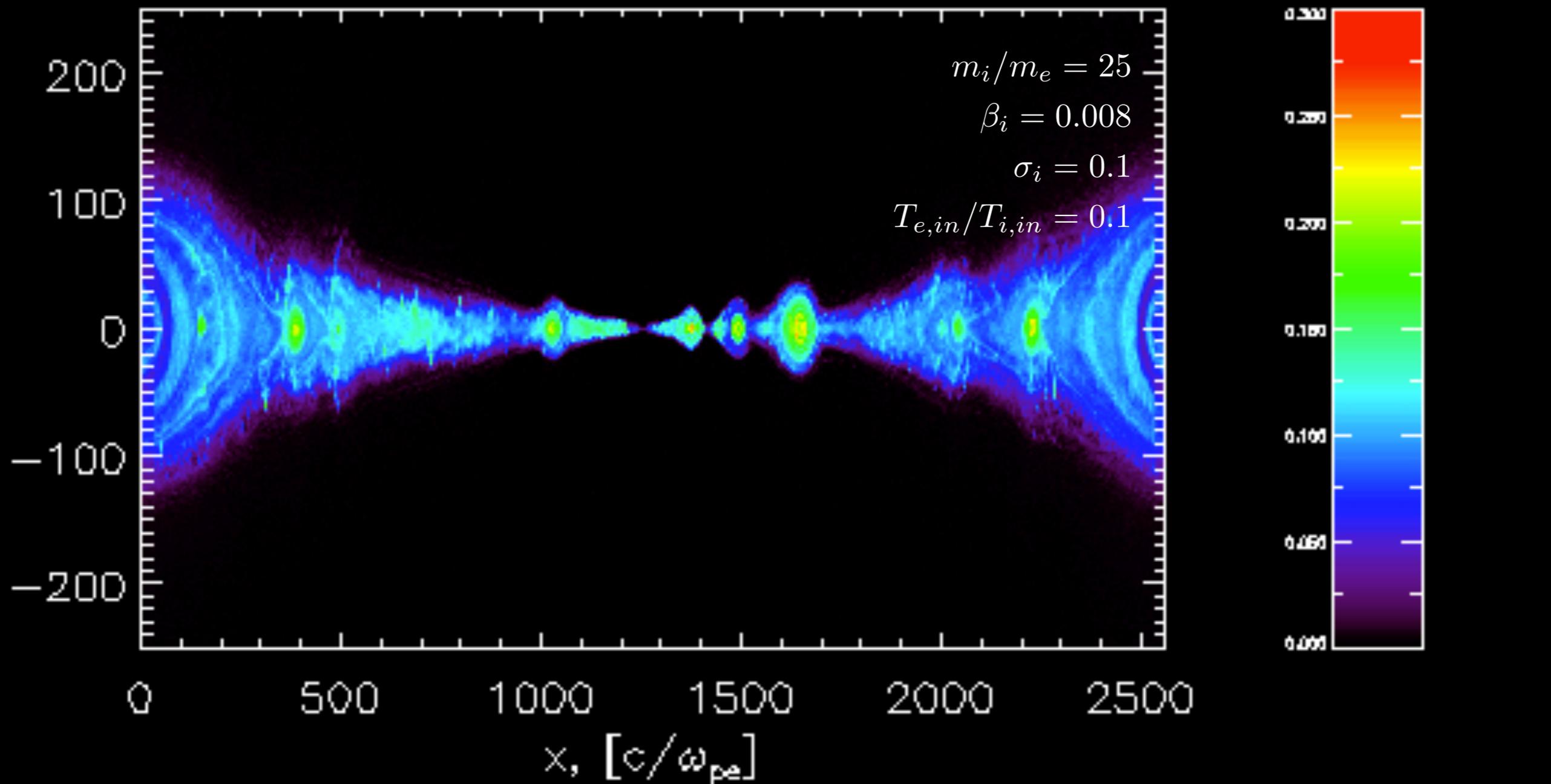
...and this is a high(er)-beta plasma

- Islands are absent; thermal pressure suppresses tearing mode
- Heuristically: the plasma is already 'hot,' so there is less heating that can be done by reconnection

$$k_b T_e / m_e c^2$$
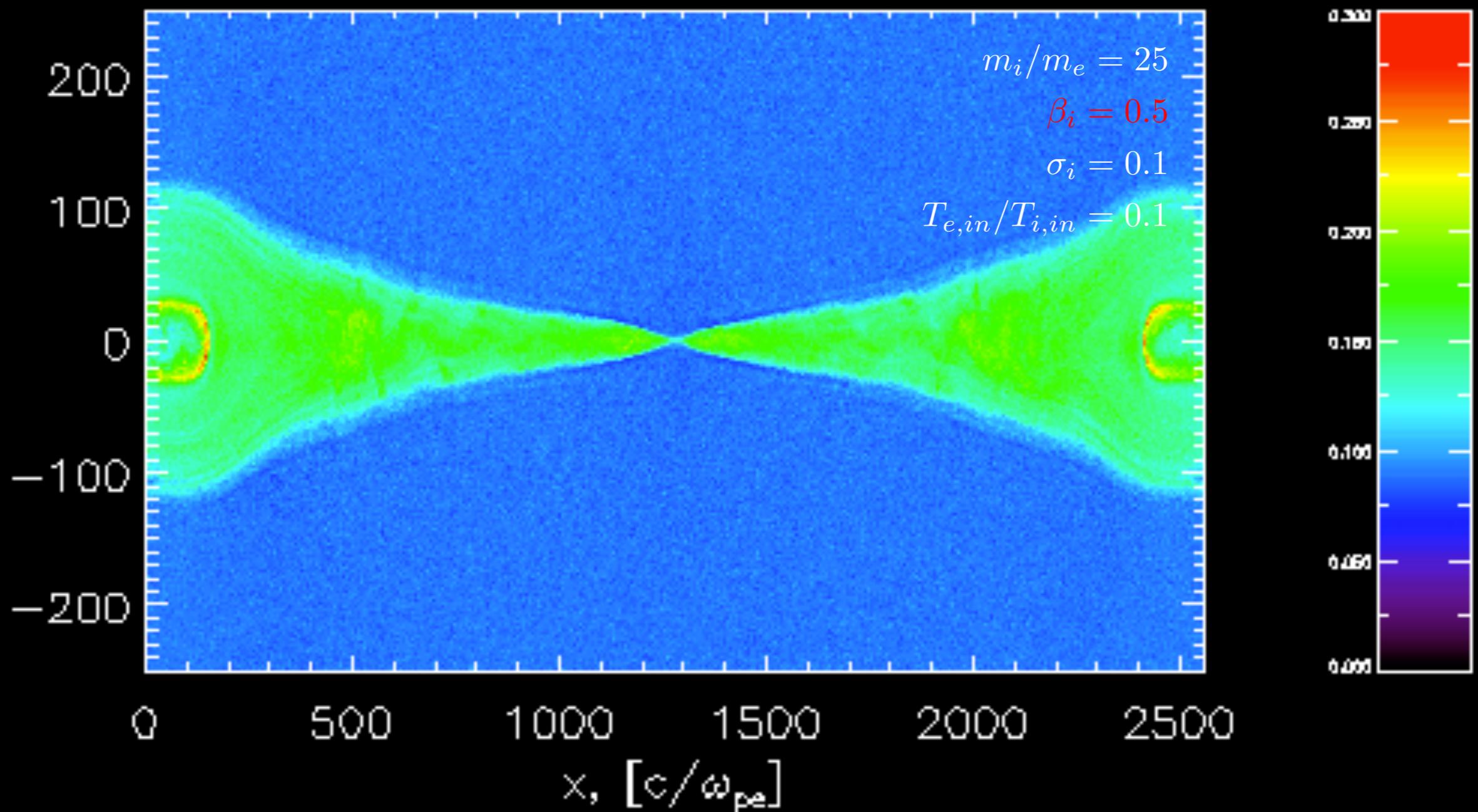
$m_i/m_e = 25$

$\beta_i = 0.5$

$\sigma_i = 0.1$

$T_{e,in}/T_{i,in} = 0.1$

$x,\ [c/\omega_{pe}]$

$$T^{\mu\nu}_{lab}$$

For each cell, compute lab-frame stress tensor

Recorded in code as:

$$\frac{1}{N_p} \sum_{particles} \frac{p^\mu p^\nu}{E}$$

We carefully extract the temperature increase

$$T^{\mu\nu}_{lab}$$

$\rightarrow$

For each cell, compute lab-frame stress tensor

$\leftarrow$

Recorded in code as:

$$\frac{1}{N_p} \sum_{particles} \frac{p^\mu p^\nu}{E}$$

$\Downarrow$

$$T^{\alpha\beta}_{CMRF} = T^{\mu\nu}_{lab} \Lambda^\alpha_\mu \Lambda^\beta_\nu$$

$\rightarrow$

Boost to fluid frame

We carefully extract the temperature increase

$$T^{\mu\nu}_{lab}$$

For each cell, compute lab-frame stress tensor

Recorded in code as:
$$\frac{1}{N_p} \sum_{particles} \frac{p^\mu p^\nu}{E}$$

$$T^{\alpha\beta}_{CMRF} = T^{\mu\nu}_{lab} \Lambda^\alpha_\mu \Lambda^\beta_\nu$$

Boost to fluid frame

$$e_0 = \rho c^2 + u_{int}$$

$$T^{\alpha\beta}_{CMRF} \stackrel{?}{=} \begin{pmatrix} e_0 & 0 & 0 & 0 \\ 0 & p_0 & 0 & 0 \\ 0 & 0 & p_0 & 0 \\ 0 & 0 & 0 & p_0 \end{pmatrix}$$

Assume perfect fluid

E.o.S:
$$(\hat{\gamma}(T) - 1)u_{int} = nT$$

Diagonal?
Isotropic?

We carefully extract the temperature increase

$T^{\mu\nu}_{lab}$ → **For each cell, compute lab-frame stress tensor** ← Recorded in code as: $\dfrac{1}{N_p} \displaystyle\sum_{particles} \dfrac{p^\mu p^\nu}{E}$

$T^{\alpha\beta}_{CMRF} = T^{\mu\nu}_{lab} \Lambda^\alpha_\mu \Lambda^\beta_\nu$ → **Boost to fluid frame**

$e_0 = \rho c^2 + u_{int}$

$T^{\alpha\beta}_{CMRF} \overset{?}{=} \begin{pmatrix} e_0 & 0 & 0 & 0 \\ 0 & p_0 & 0 & 0 \\ 0 & 0 & p_0 & 0 \\ 0 & 0 & 0 & p_0 \end{pmatrix}$ → **Assume perfect fluid** ← E.o.S: $(\hat{\gamma}(T) - 1)u_{int} = nT$

Diagonal? Isotropic?

Iterate E.o.S. and adiabatic index eq. → Temperature
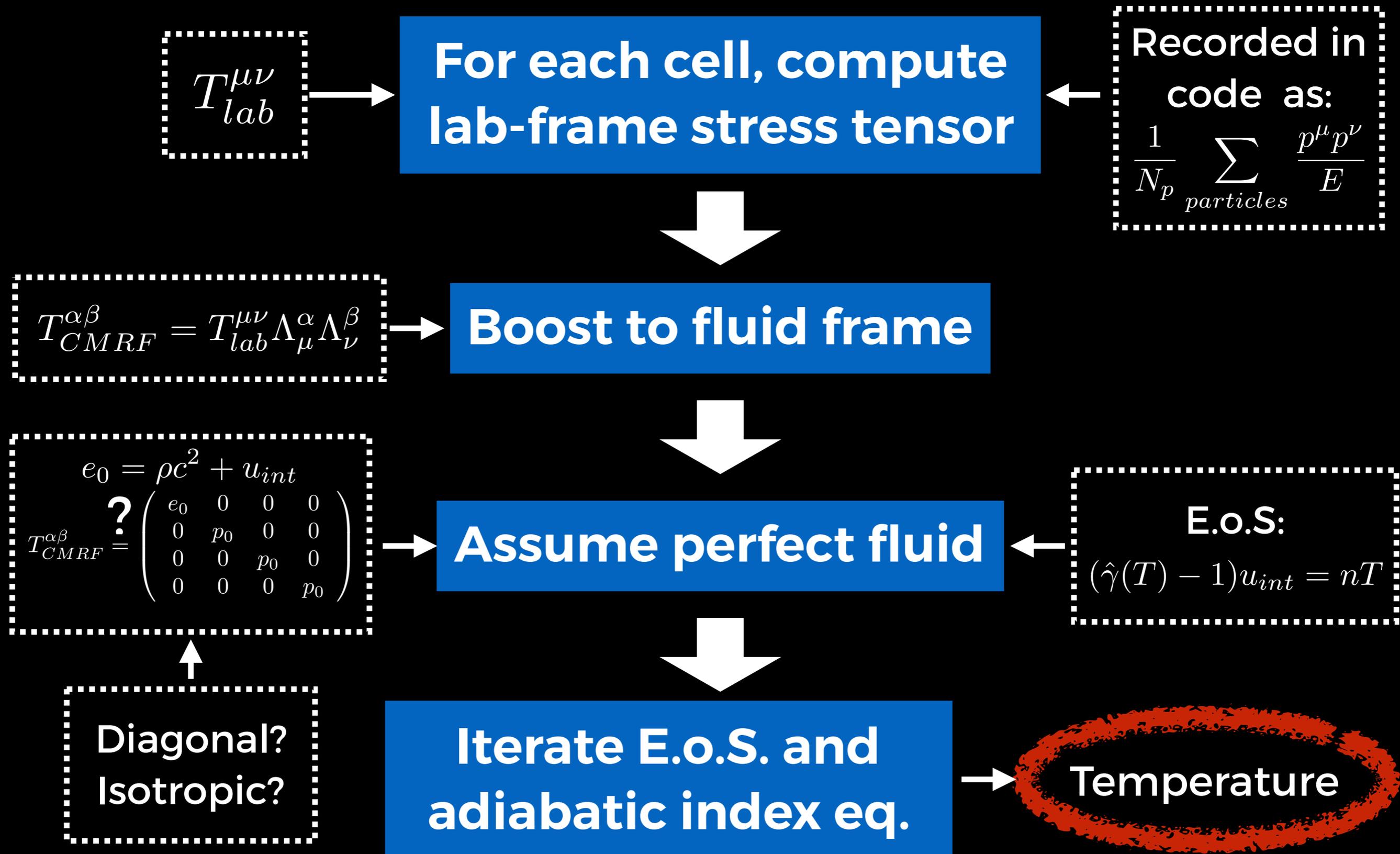
Electron gross heating

- For low-beta, the fraction of magnetic energy that ends up as electron heating is around 3%
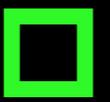- Dependence on initial temperature ratio
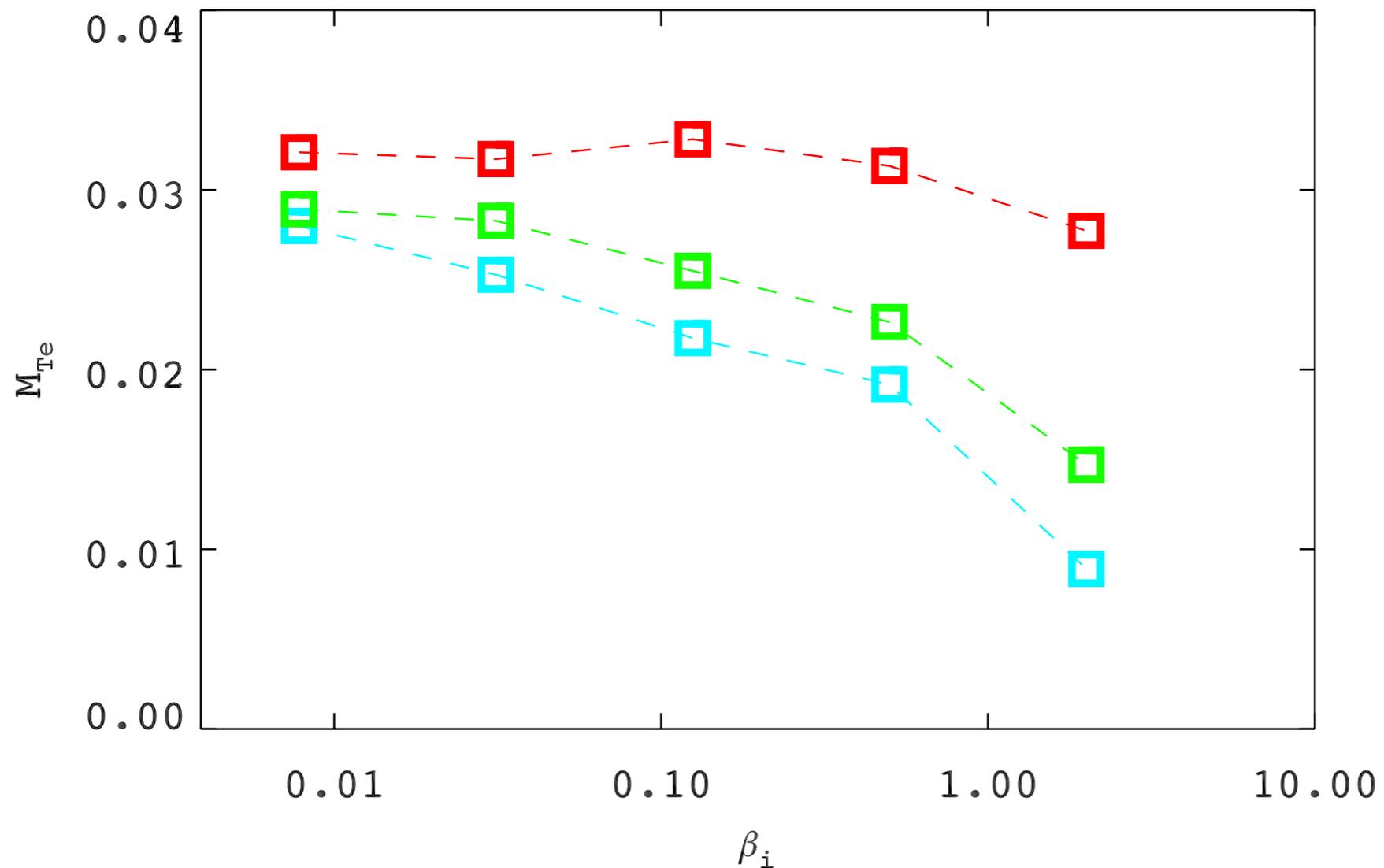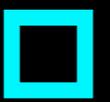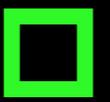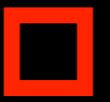
Ion gross heating
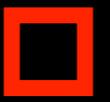
- Free magnetic energy that ends up as ion heating: ~10-12%
- This gives us a rough value for electron:ion heating as 1/3 in the low-beta cases
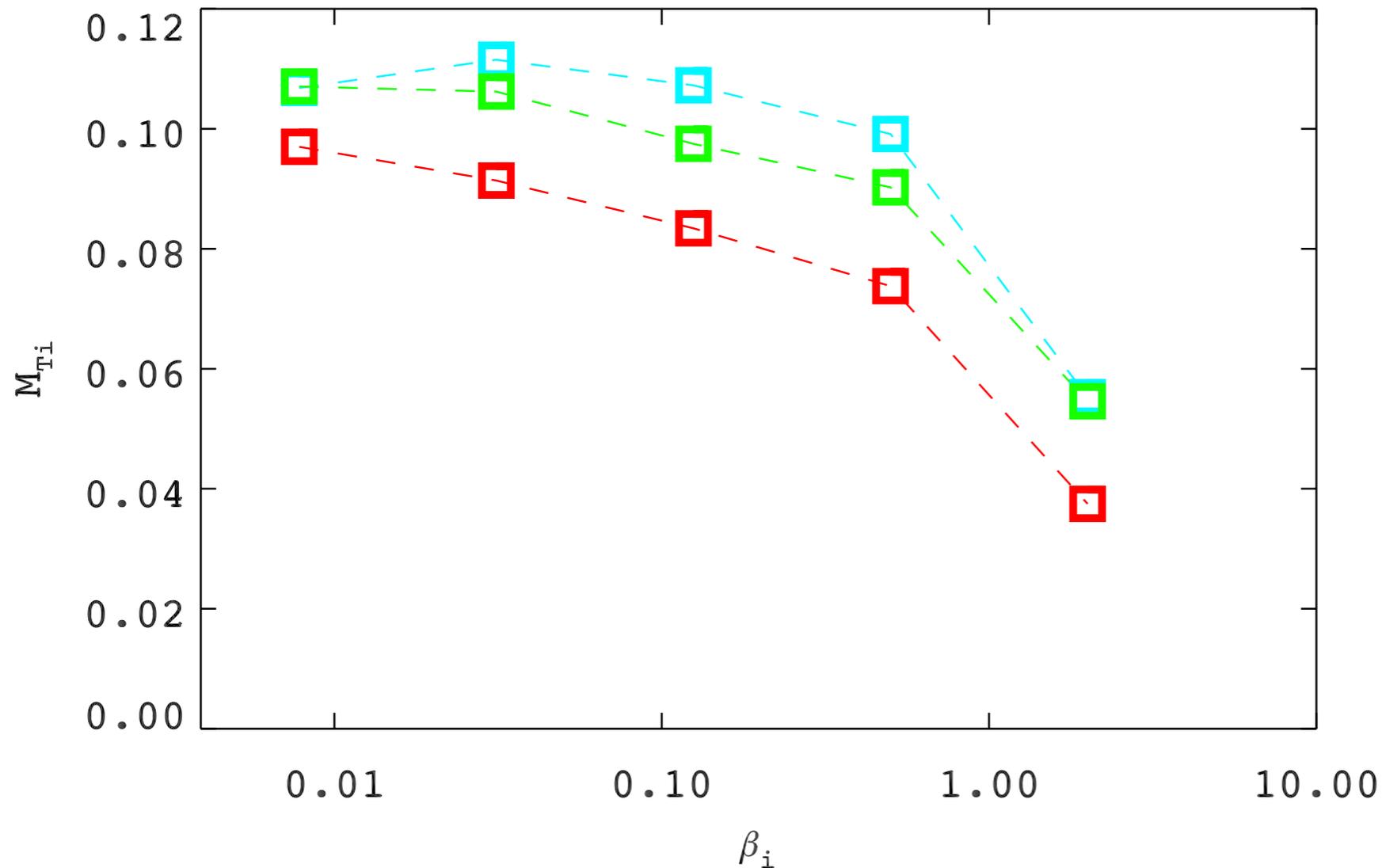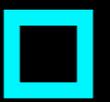
Compressive vs. non-compressive heating

▷ We use the following equation of state to account for the fact that electrons' adiabatic index can vary from beginning to end of reconnection:
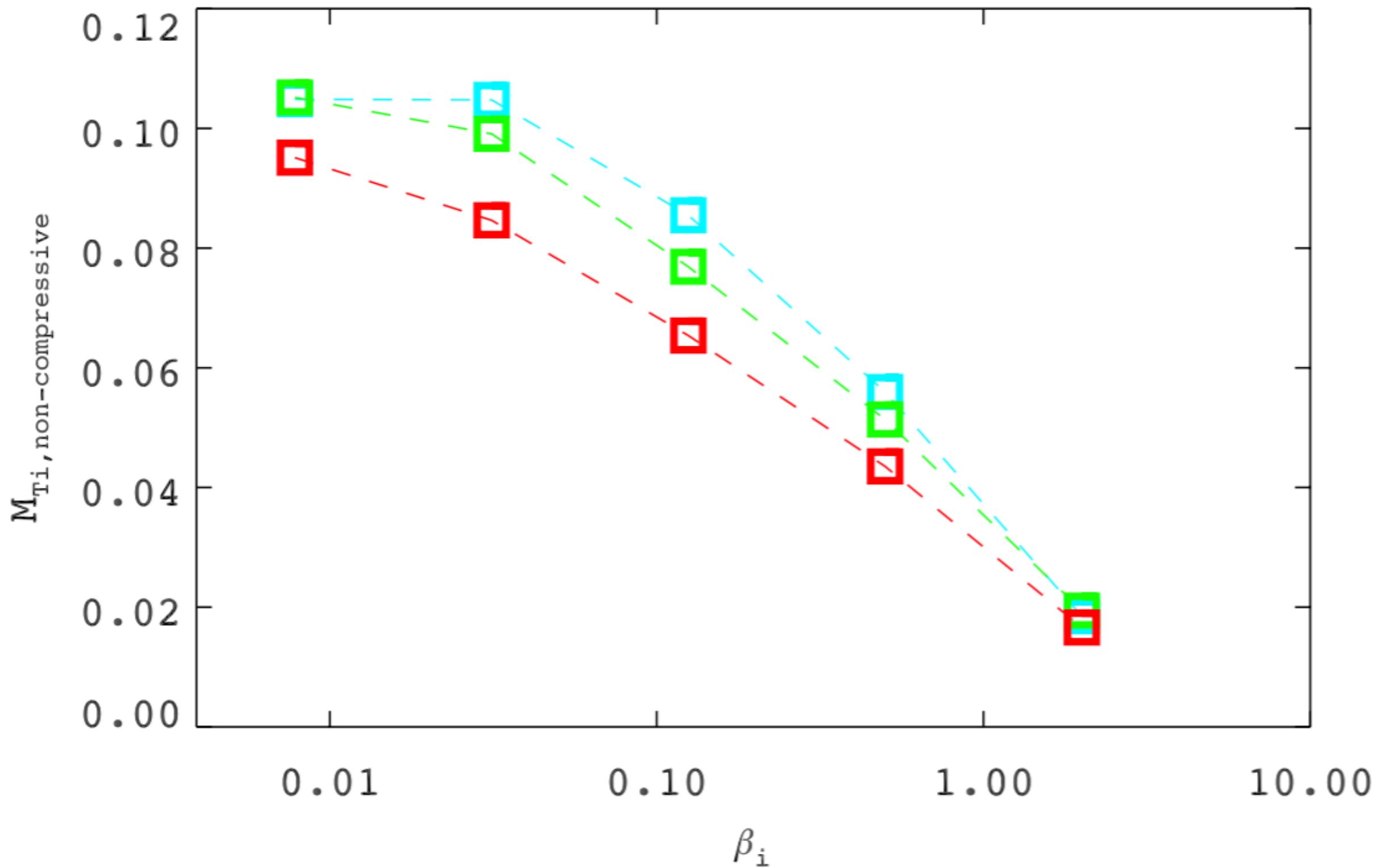
Equation of state for variable adiabatic index

$$const = \frac{p}{\rho^{5/3}} \left(\frac{3}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1} \right), \Theta = \frac{kT}{mc^2}$$

▷ This allows us to remove the compressive component of the heating, which is not a result of heating due to the reconnection electric field
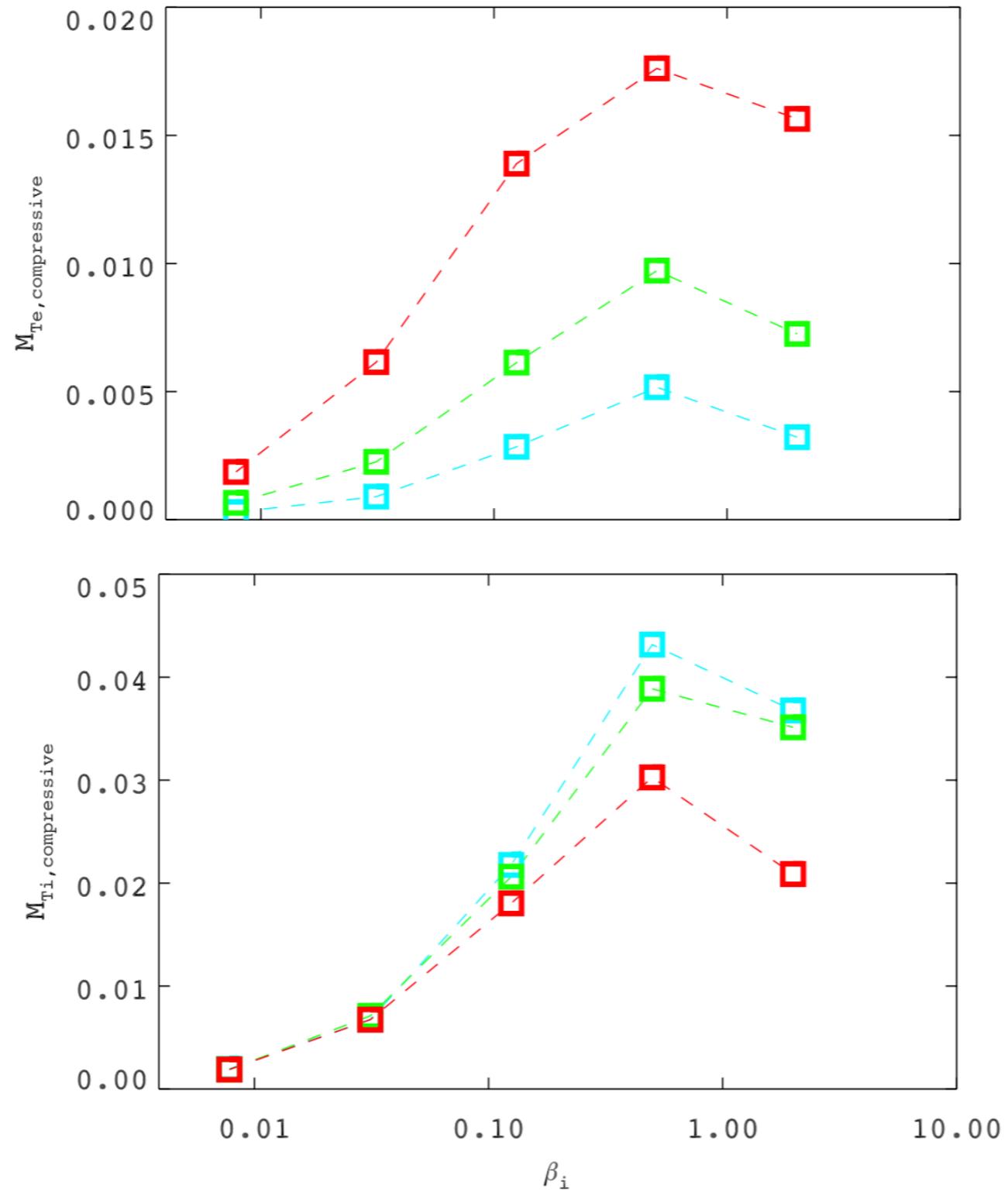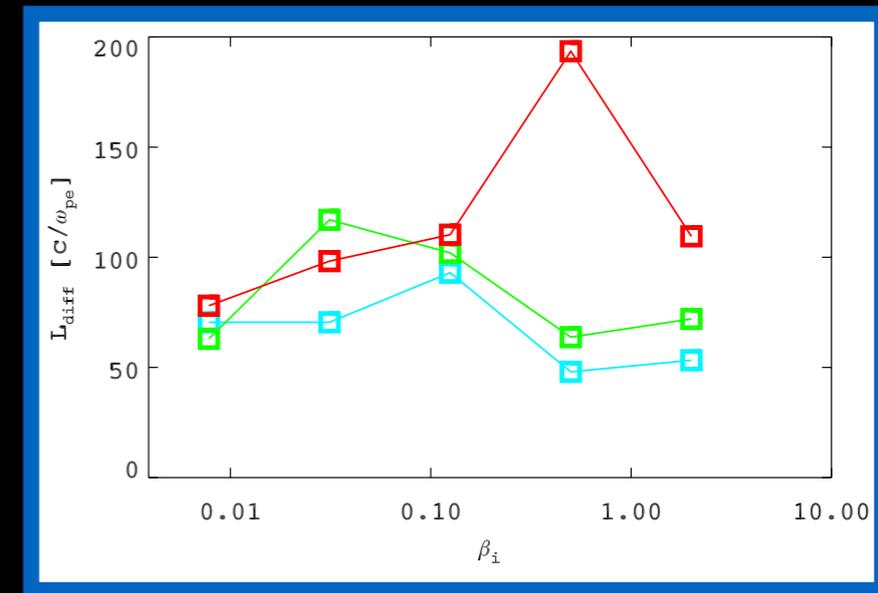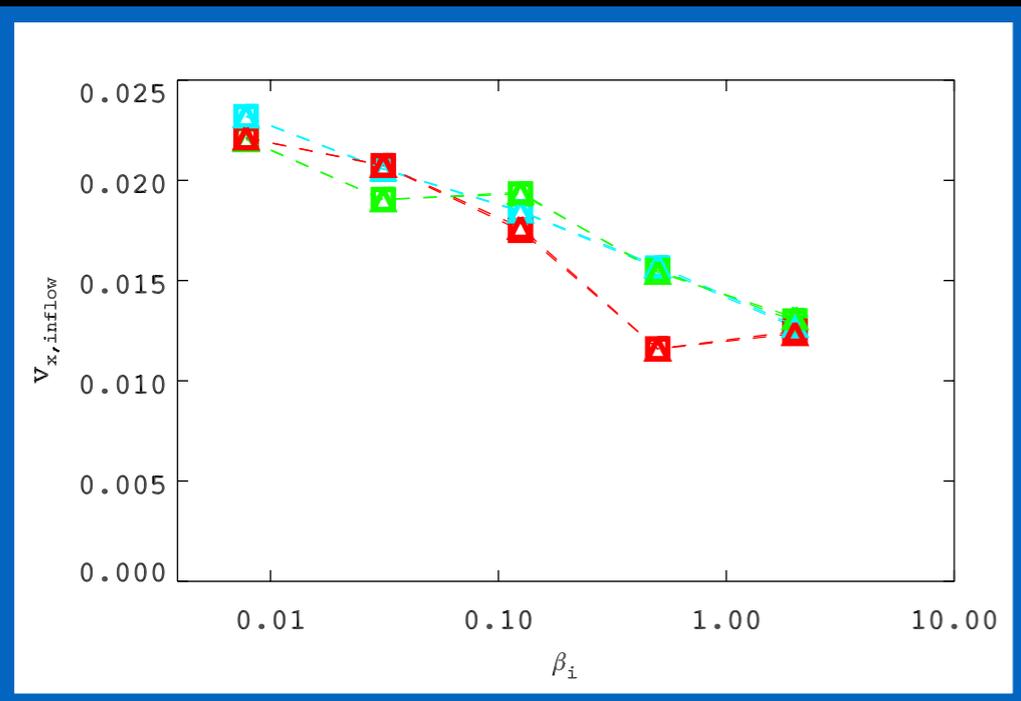
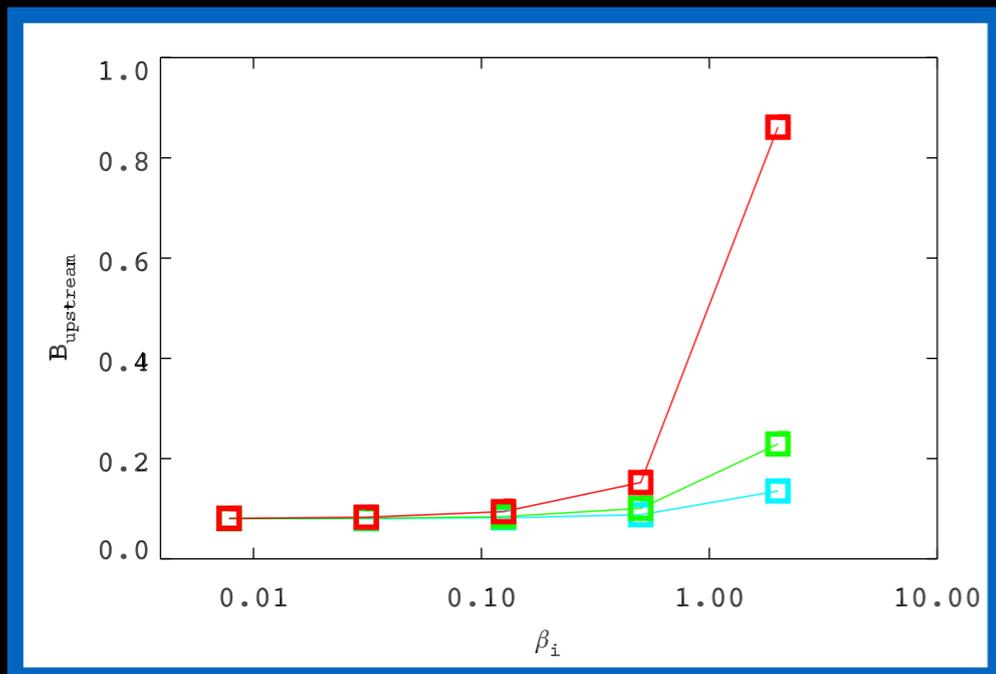# Electrons: heating analysis
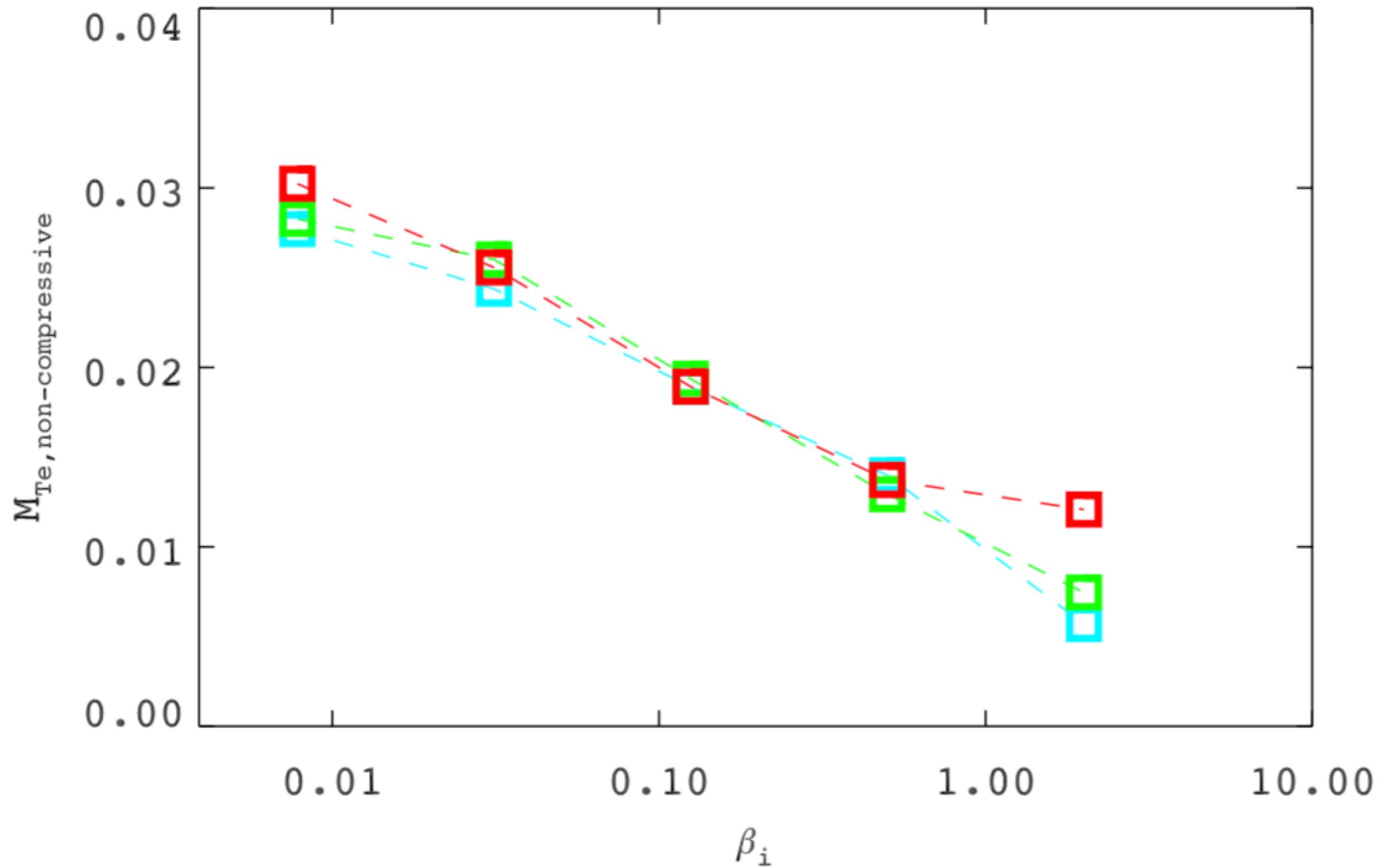
Ions: heating analysis

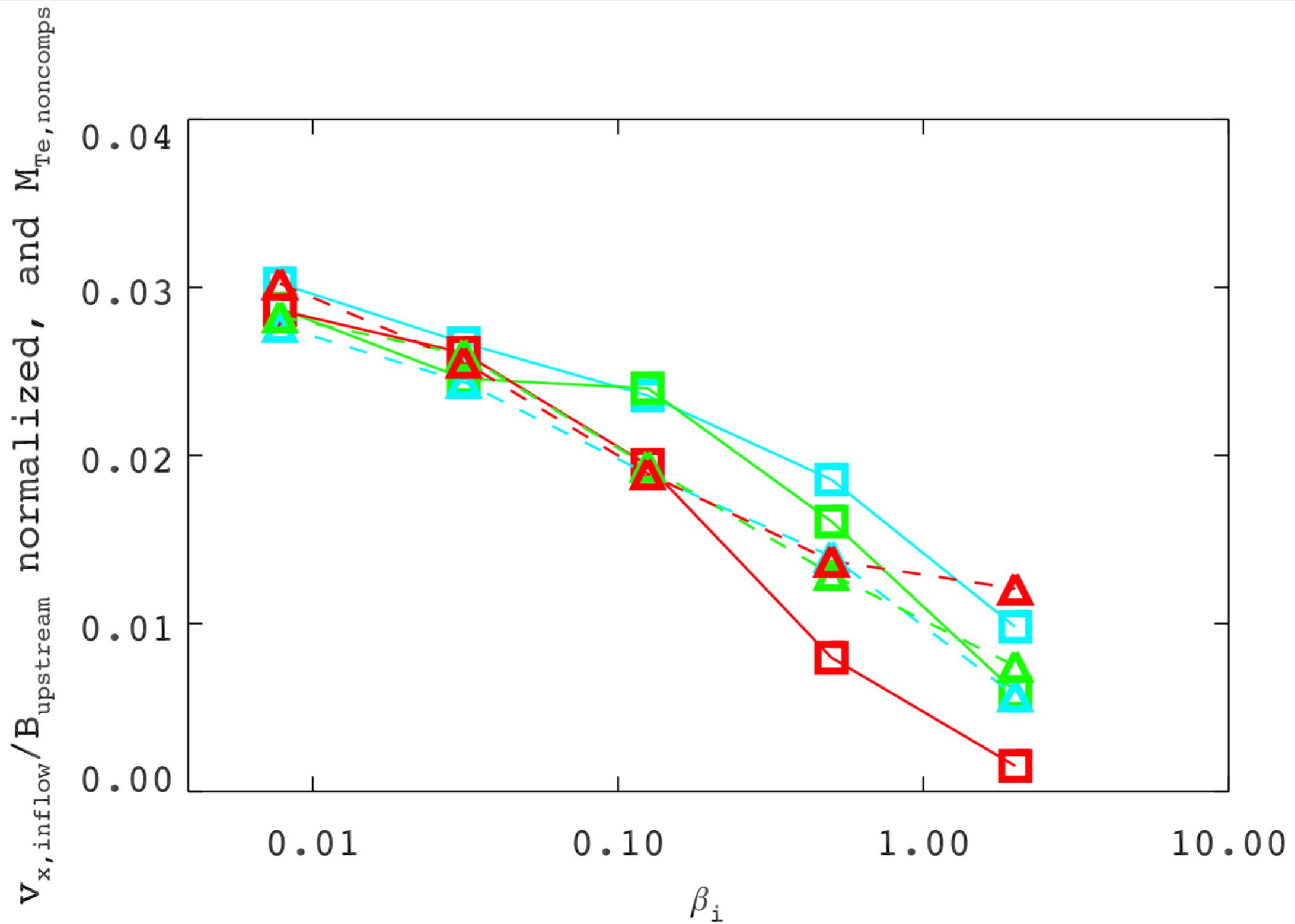# Compressive heating

A simple model for the electron heating

$$M_{Te,ideal} \sim \frac{eE_{rec}Ln}{B^2/8\pi} \sim \frac{e\left(\frac{v_{in}}{c}\right)Ln}{B}$$

▷ The expression is roughly the work done by reconnection E field compare to inflow magnetic energy
▷ Treat B, L, and v_{in} as functions of beta, T_e/T_i - don't just assume constant

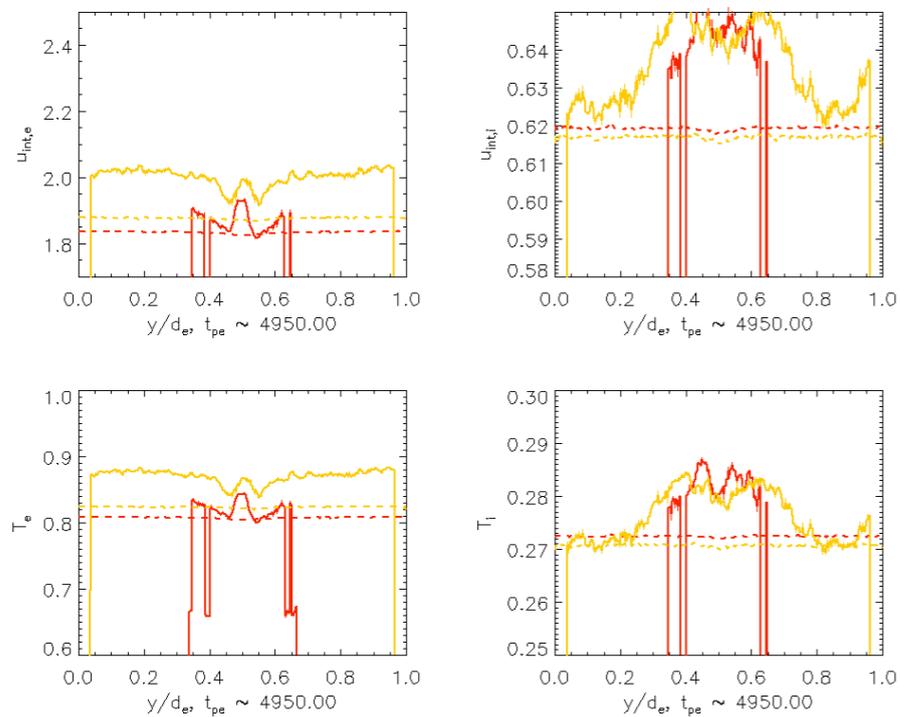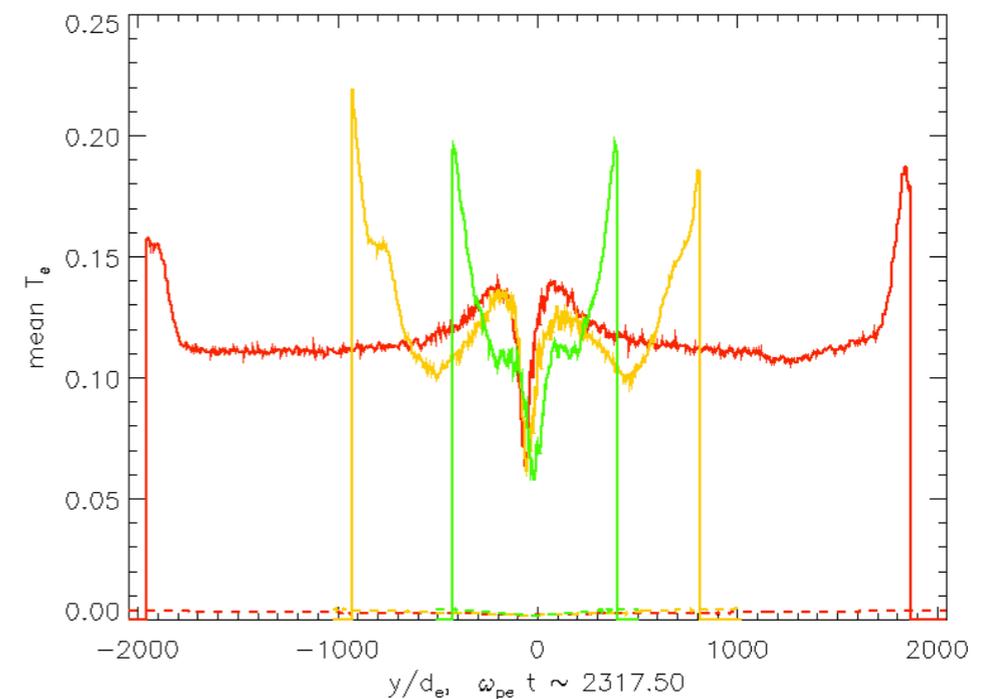# Heating should not depend on nonphysical parameters

- Check for convergence by varying computational parameters
- To trust the numerical results, need to make sure numerical heating is relatively small
- Particles per cell, domain size, boundary conditions, etc.

ppc = 64 (yellow), ppc = 256 (red)
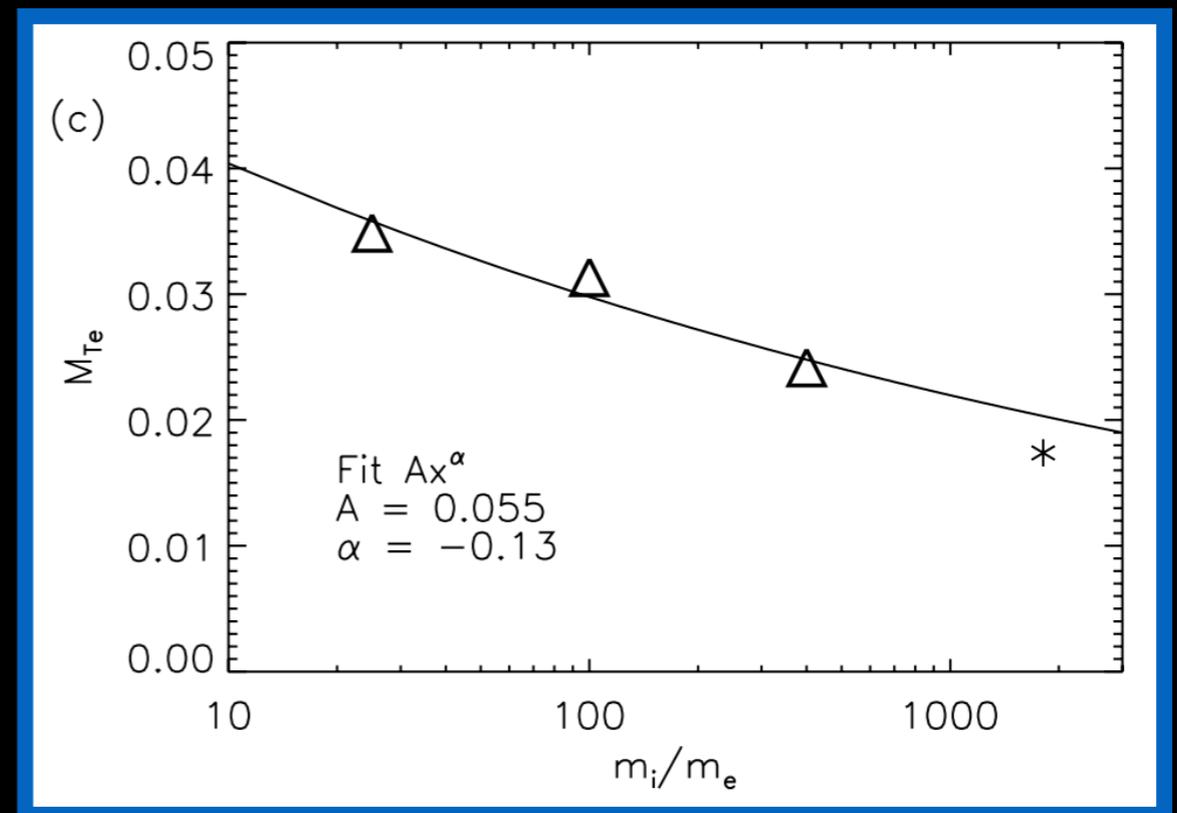
m_y = 4096 (green), 8192 (yellow), 16384

Electron heating will decrease with higher m_i/m_e

▷ In our simulations, we use an artificial mass ratio of $m_i/m_e = 25$
  ▷ Why? This makes the problem computationally tractable
▷ We can expect our measured heating will decrease with higher mass ratio;

$$M_{Te} \sim (m_i/m_e)^{-0.13}$$

(Drake et al., 2014)

▷ Note: this scaling is consistent with the analytical model of Egedal et al.

Connection to black-hole physics

- Two main aspects to our investigation
  - Plasma physics
    - Explore a relatively unstudied region of plasma parameter space
  - Astrophysics
    - Provide (eventually) a lookup table for global simulations of black-hole accretion flows
    - Even if it turns out that the dependence on input values is weak, at least this will be known from a first-principles investigation

Summary:

▷ Reconnection provides less net heating for high-beta compared to low-beta; $T_{e,\,out}\,/\,T_{e,in}$ approaches 1 for high beta

▷ Low-beta: ~3% of the magnetic energy ends up as electron heating, and ~10-12% ands up as ion heating

For the future:

▷ Explore guide field reconnection

▷ Push to higher beta

▷ Vary the mass ratio

▷ Run with wider range of sigma

▷ Use particle orbits to study heating mechanism

    ▷ Is this the same as in the non-relivistic case?

▷ 3D simulations

Thank you for your attention

Strange-looking point from plot of L(beta)
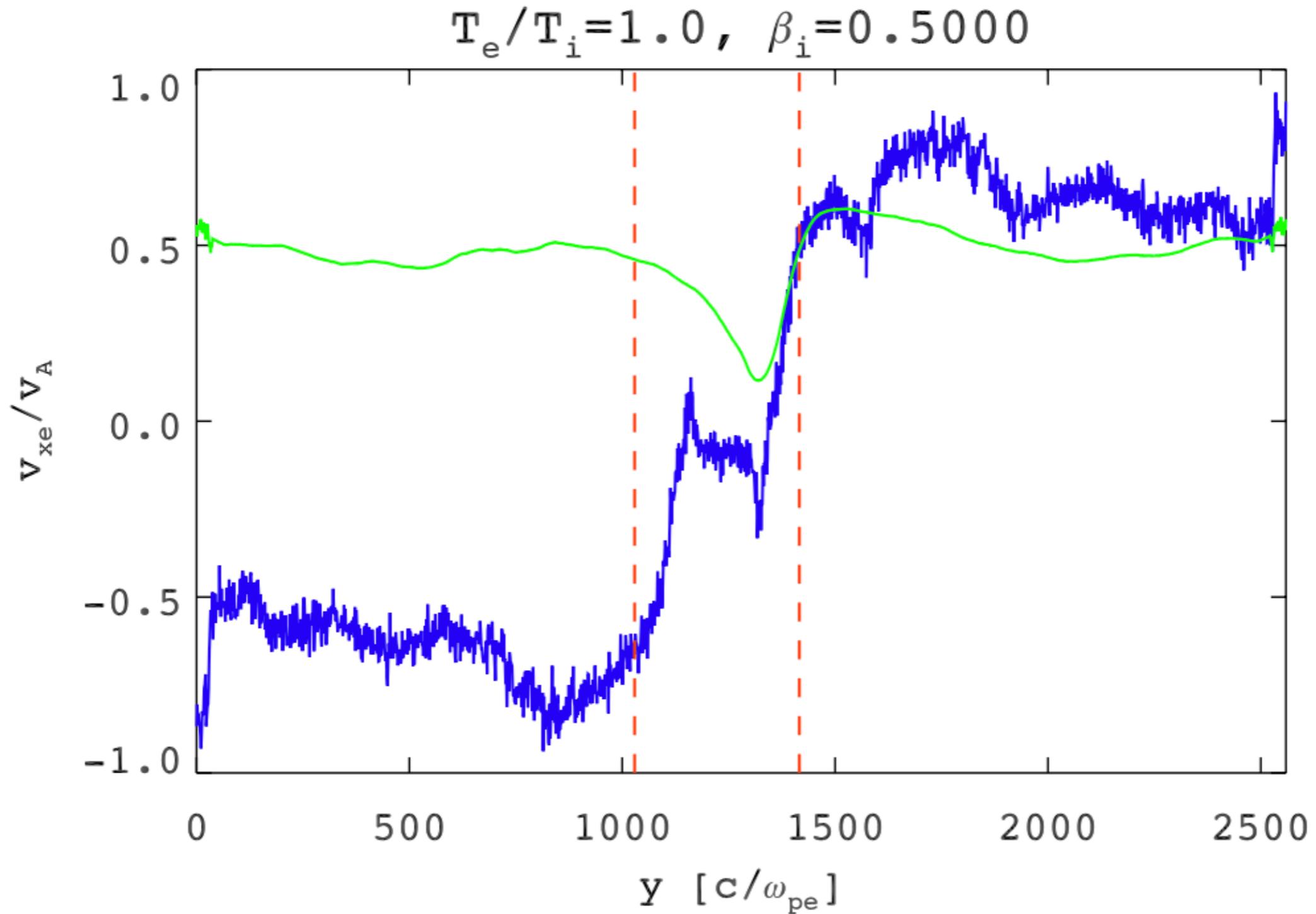
E.o.S. derivation

What is our equation of state if the adiabatic index is not constant?

$$dS = \frac{dQ}{\Theta}, \, dQ = dH - V\,dp, \, \Theta = \frac{p}{\rho}$$

$$\Rightarrow dS = \frac{dH}{\Theta} - V\frac{dp}{\Theta} = \frac{dH}{\Theta} - \frac{N}{p}dp$$

$$dS = \frac{dh}{\Theta} - d\log p$$

Now integrate, using equation for $h(\Theta)$:

$$h = \frac{5}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}$$

$$\Rightarrow \int dS = \int \frac{1}{\Theta}\left(\frac{5}{2} + \left(\frac{9}{4}\Theta^2 + 1\right)^{-1/2}\frac{9}{4}\Theta\right)d\Theta - \log p$$

$$S = \frac{5}{2}\log\Theta + \frac{3}{2}\sinh^{-1}\left(\frac{3}{2}\Theta\right) - \log p$$

$$= \frac{5}{2}\log\Theta + \frac{3}{2}\log\left(\frac{3}{2}\Theta + \sqrt{\left(\frac{3}{2}\Theta\right)^2 + 1}\right) - \log p$$

$$= \frac{3}{2}\left(\log\Theta + \log(h - \Theta)\right) + \log\left(\frac{\Theta}{p}\right)$$

$$= \frac{3}{2}\log\left(\frac{\Theta^{5/3}}{p^{2/3}}(h - \Theta)\right)$$

$$= \frac{3}{2}\log\left(\frac{p}{\rho^{5/3}}(h - \Theta)\right)$$

$$const = \frac{p}{\rho^{5/3}}\left(\frac{3}{2}\Theta + \sqrt{\frac{9}{4}\Theta^2 + 1}\right)$$

This provides the correct limiting values. For $\Theta \to 0$,

$$const = \frac{p}{\rho^{5/3}}$$

and for $\Theta \gg 1 \Leftrightarrow p \gg \rho$,

$$const \simeq \frac{p}{\rho^{5/3}}3\Theta$$

$$\Rightarrow const' = \frac{p^2}{\rho^{8/3}}$$

$$\Rightarrow const'' = \frac{p}{\rho^{4/3}}$$

The adiabatic index is not the same in the upstream as compared to outflow region. To make a meaningful comparison of the compressive heating, we should compute $const$ upstream, then use the variable equation of state (boxed above) to compute the predicted value $const$ for given Θ, p, ρ in the outflow. Discrepancy between the predicted and actual values should then be accounted for by 'actual' heating.

The equation we used for specific enthalpy comes from Taub inequality, taking the equals sign:

$$(h - \Theta)(h - 4\Theta) \geq 0$$

$$\Rightarrow h^2 - h5\Theta + 4\Theta^2 - 1 \geq 0$$

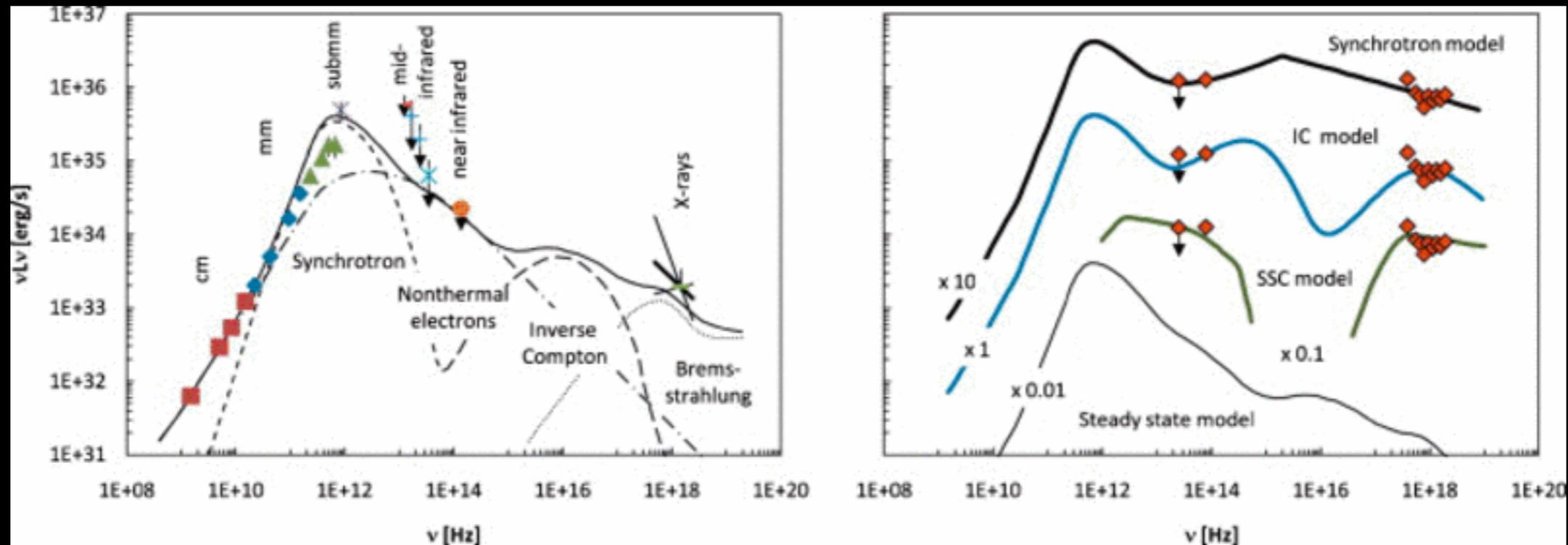$$\Rightarrow h = \frac{5}{2} + \sqrt{\frac{9}{4}\Theta^2 + 1}$$

Figure 30(Color) Spectral energy distribution of Sgr A*. All numbers are given for a distance of 8.3kpc of the Galactic Center and are dereddened for interstellar absorption (infrared and x rays) and scattering (x rays). Left: steady state. The Sgr A* radio spectrum follows roughly a power law $\nu L_\nu \sim \nu^{4/3}$. The observed peak flux at submillimeter wavelengths is about 5×10^{35}erg/s. The spectrum then steeply drops toward infrared wavelengths down to less than the detection limit of about 2×10^{34}erg/s at 2μm. The only other unambiguous detection of Sgr A* in its steady state is at x rays with energies from 2−10keV with a flux of about 2×10^{33}erg/s. The figure shows a compilation of data (with increasing frequency) from ■ (560), ◆ (150), ▲ (565), × (487), - (105), + (186), × (475), ● (243), and − (34). Overplotted is a model of the quiescent emission [adapted from 541]: the radio spectrum is well described by synchrotron emission of thermal electrons (short-dashed line). The flattening of the radio spectrum at low frequency is modeled by the additional emission from a nonthermal power-law distribution of electrons, which carry about 1.5% of the total thermal energy (dash-dotted line). The quiescent x-ray emission arises from thermal bremsstrahlung from the outer parts of the accretion flow (dotted line). The secondary maximum (long-dashed line) at frequencies of about 10^{16}Hz is the result of the inverse Compton upscattering of the synchrotron spectrum by the thermal electrons. Right: SED during a simultaneous x ray and infrared flare: while the total

1. Solve for the fields

• In electromagnetic PIC codes, only two equations need to be solved.

$$\partial_t B = -\nabla \times E,$$
$$\partial_t E = \nabla \times B - J.$$

• The other two are satisfied as initial conditions, and they continue to be satisfied for appropriate choices of the numerical scheme.
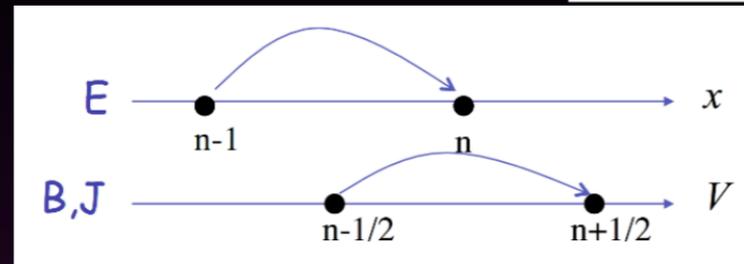
$$\nabla \cdot E = \rho,$$
$$\nabla \cdot B = 0.$$

STAGGERING in time (leapfrog):

• second-order accurate in time

$$E^{n+1/2} = E^{n-1/2} + \Delta t[c(\nabla \times B^n) - 4\pi J^n]$$
$$B^{n+1} = B^n - c\Delta t \nabla \times E^{n+1/2}.$$

STAGGERING in space (Yee's mesh):

• electric fields on cell edges, magnetic fields on cell faces

• second-order accurate in space

• maintains divergence-free B

$$\partial_t B = -\nabla \times E,$$
$$\partial_t E = \nabla \times B - J$$
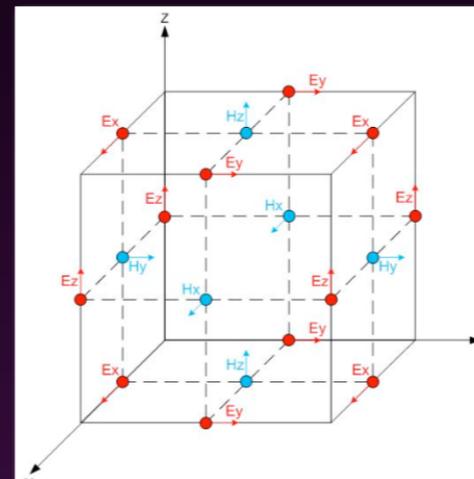
- The fields obtained from Maxwell's equations are determined only at the grid points, they need to be interpolated to the particle positions.

- The interpolation is done by assuming a particle shape function.

- The shape function needs to be:

  1. isotropic

  2. zero outside some range

  3. higher order B-splines are computationally more expensive, but more accurate and less "collisional"
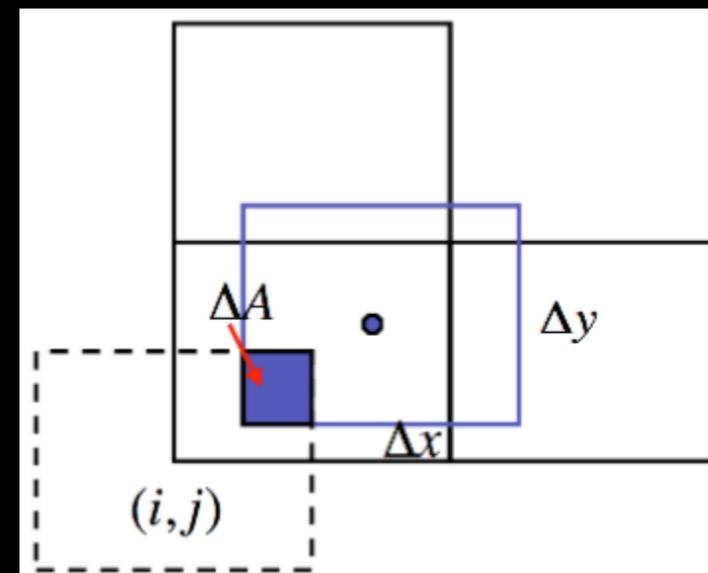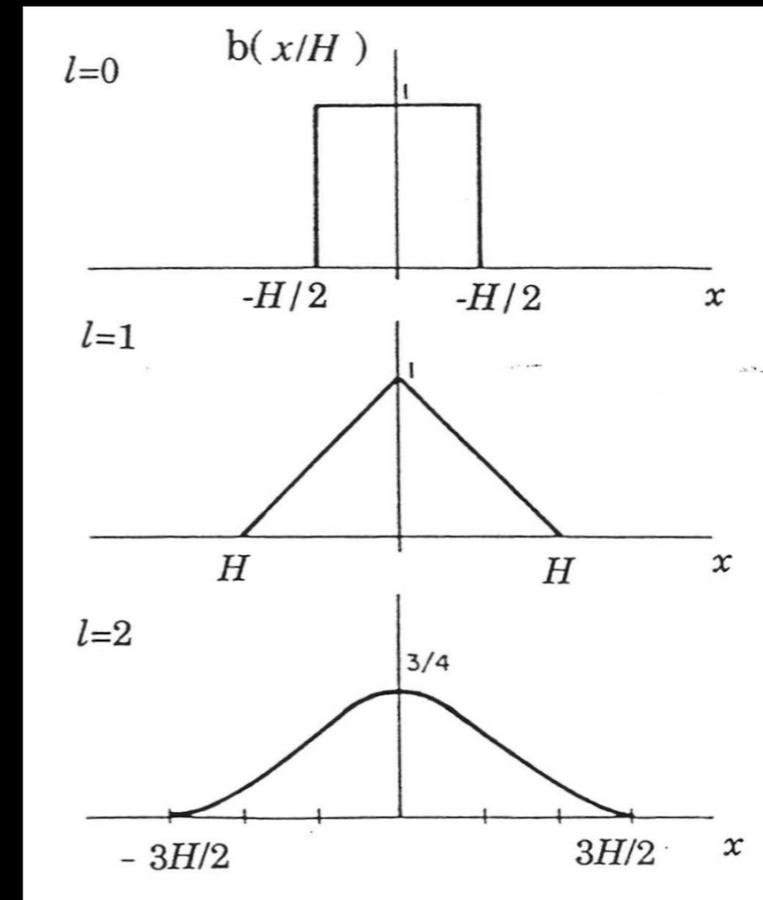
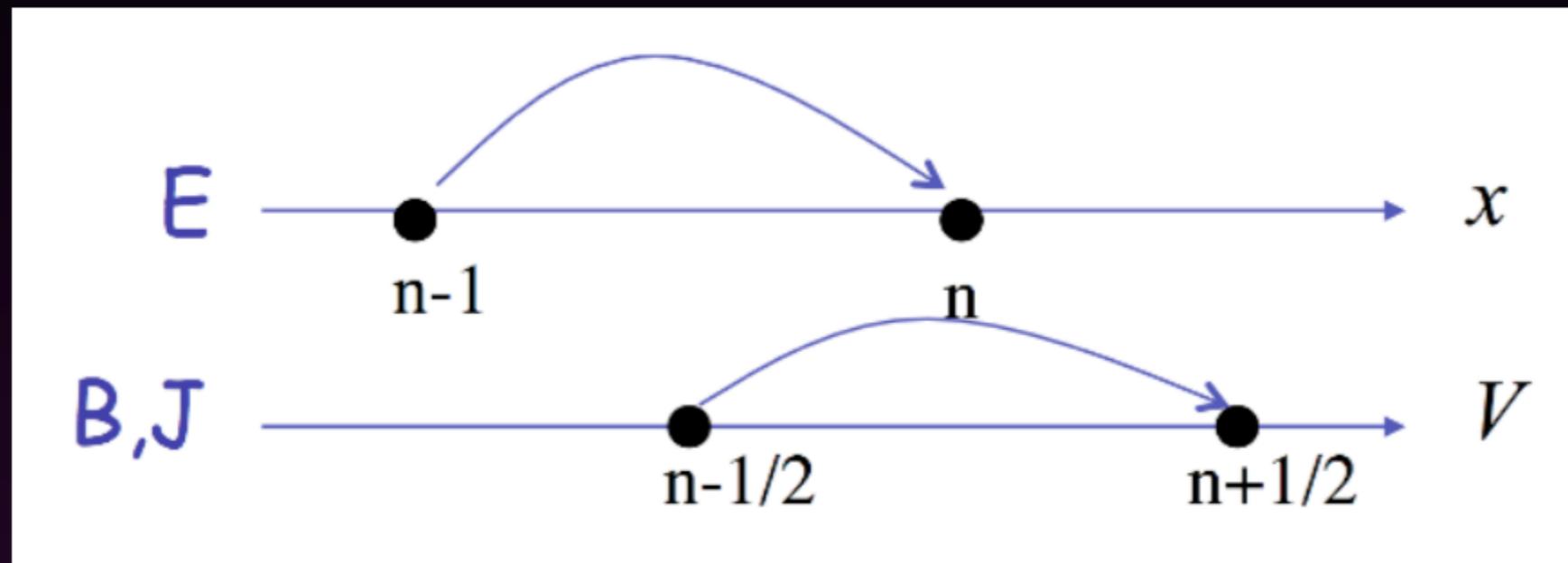$$E(\vec{x}_k) = \sum_{i,j} E_{ij} S(\vec{x}_k - \vec{X}_{ij})$$

$$S(\vec{x}_k - \vec{X}_{ij}) = \frac{\Delta A}{\Delta x \Delta y}$$

$$\sum_{i,j} S(\vec{x}_k - \vec{X}_{ij}) = 1$$

3. Push the particles

If the number of ppc is >>1, most of the computing time is spent in pushing the particles.

The BORIS pusher (leapfrog method)

advance the position

$$x^{n+1} = x^n + \frac{p^{n+1/2}}{\gamma^{n+1/2}} \Delta t$$

advance the momentum

$$\frac{p^{n+1/2} - p^{n-1/2}}{\Delta t} = \frac{q}{m}(E^n + \frac{1}{c}\frac{p^{n+1/2} + p^{n-1/2}}{2\gamma^n} \times B^n)$$
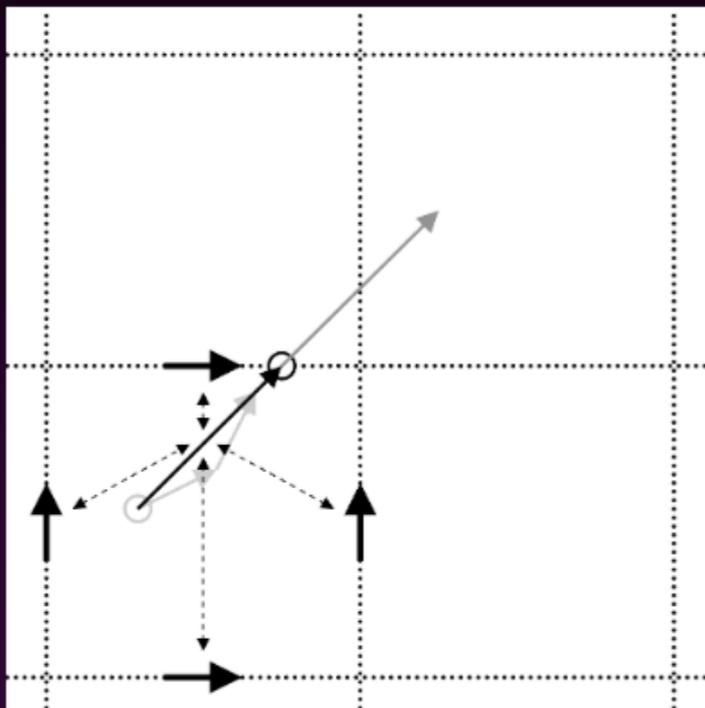
$$(B^n = \frac{B^{n+1/2} + B^{n-1/2}}{2})$$

• Charge conservation is required to satisfy Poisson's equation.

$$\partial_t B = -\nabla \times E \Rightarrow \partial_t \nabla \cdot B = 0$$

$$\partial_t E = \nabla \times B - J \Rightarrow \partial_t \nabla \cdot E = -\nabla \cdot J \xrightarrow{\ ?\ } \partial_t \rho$$

• The current deposition scheme needs to be charge-conserving.
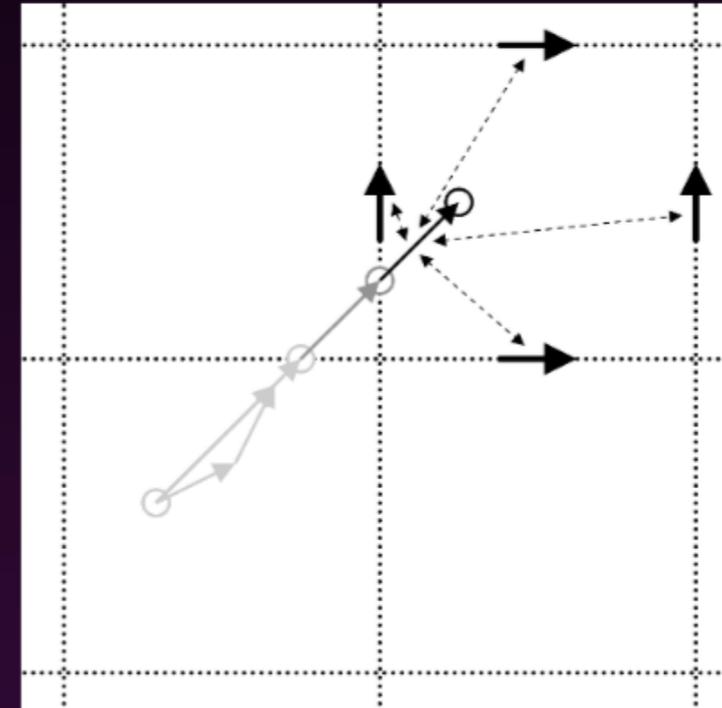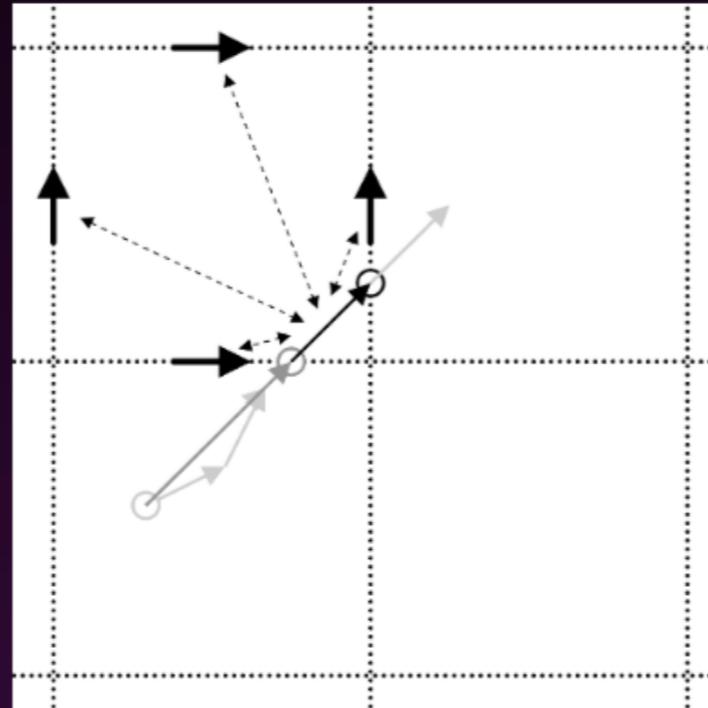
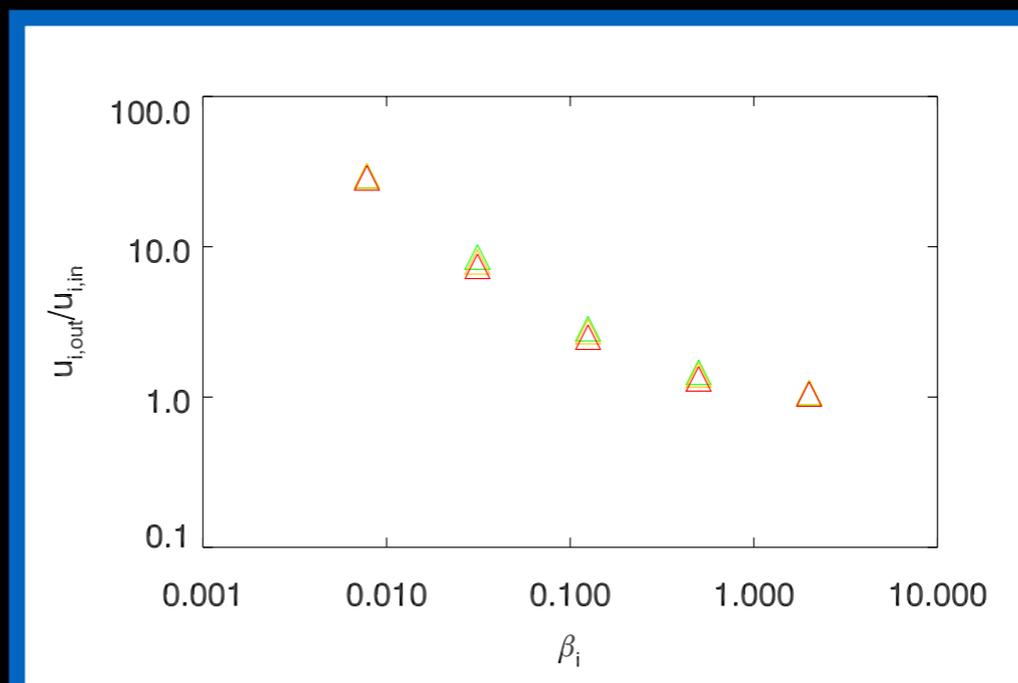• Or, a divergence-cleaning solver should be employed.
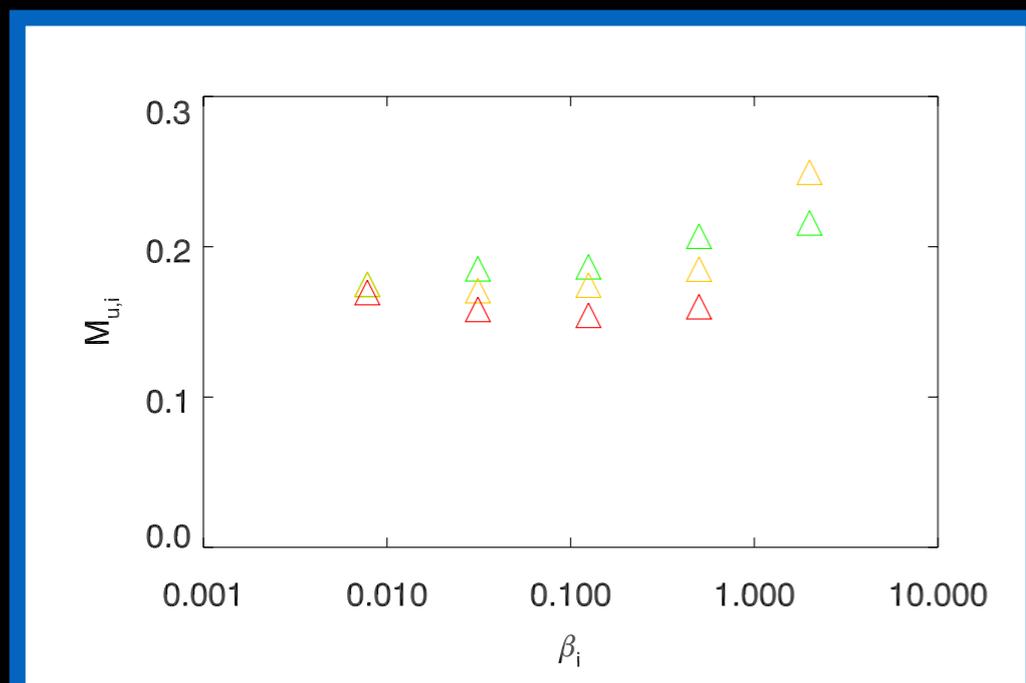
Numerical stability

- The particle granularity gives short-scale fluctuations of the electromagnetic fields, whose mean amplitude scales (Poisson-like) as $\sqrt{n}$, where n is the particle density.

- The fractional contribution of the fluctuations (over the slowly varying fields) scales as $1/\sqrt{n}$.

- This is problematic because the number of super-particles in particle-in-cell codes is $\ll$ number of real particles.

- We need to control the level of the fluctuations such that they give negligible effects over the timespan of the simulations.

Electron vs. Ion heating